Tomas Bata University in Zlín
Faculty of Applied Informatics

Eric Afful-Dadzie, Msc.

Hybridized Integrated Methods in Fuzzy Multi-Criteria Decision Making

(With Case Studies)

Doctoral thesis

Course:	Engineering Informatics
Selected field:	Engineering Informatics
Supervisor:	assoc. prof. Zuzana Komínková Oplatková, Ph.D

Zlín, 2015

DEDICATION

This dissertation is dedicated to the two greatest women in my life; *Naana Mensima,* my wife and *Auntie Mary,* my mum. It is also dedicated to my two lovely boys; Jason and Ethan. May this piece of work be an inspiration for them to achieve greater feat than their daddy.

It is further dedicated to my in-laws *Pet* and *Oman* for holding the fort in my long absence to pursue further studies. I am extremely grateful for shouldering such great responsibilities of providing warmth and love to my family. *I will always have you in my thoughts*.

It is also dedicated to my 'twin' brother Ato and the entire family; Nana Otu, Maame Esi, Maame Ekua, and Magdalene. May God richly bless you all.

ACKNOWLEDGEMENTS

My doctoral studies would never have been possible without the immense support from many individuals. In particular, I acknowledge the contributions of my supervisor, assoc. prof. Zuzana Komínková Oplatková, who was always helpful and supportive when I most needed it. I am also grateful to her for reading through my numerous revisions and helping to make this dissertation a reality. I cherished the constructive criticisms and commentaries on my work.

I also acknowledge the love and support I received from prof. Ing. Roman Prokop, CSc., especially during some of the most difficult times at the beginning of my studies. I would like to thank him for his understanding and care.

Special gratitude goes to my brother, Dr. Anthony Afful-Dadzie who served as my most cherished research partner in most of my publications. I am grateful to him for his numerous reviews, commentaries, arguments and editing of my works. Significant recognition also goes to several brilliant friends and colleagues I came across in Zlin especially those who in diverse ways contributed to my studies and life in Zlin. Special mention goes to Ing. Stephen Nabareseh, Ing. Michael Adu-Kwarteng, Carlos Beltrán Prieto, Ph.D, Bc. Jana Doleželová, Ing. Tomáš Urbánek, and Ing. Stanislav Sehnálek, for their diverse contributions to my study in Zlin.

Also noteworthy of mention are the Dean, assoc. prof. Milan Adámek, Ph.D., the head of department of Informatics and Artificial Intelligence, assoc. prof. Mgr. Roman Jašek, Ph.D. and assoc. prof. Ing. Roman Šenkeřík.

Last but not the least, I would also like to recognize and extend my appreciation to the committee members, for taking time to read the dissertation and offering constructive criticisms that helped improved the final work.

ABSTRACT

Multi-criteria decision making (MCDM) under fuzzy settings has been utilized in many wide ranging applications in industry and academia. However, a current trend in several of these works, is the use of more than one MCDM method in ranking and selection problems. For example, in a supplier selection problem, the Analytical Hierarchy Process (AHP) or the Analytic Network Process (ANP) method may be used to set the weights of the criteria and a different MCDM method used to rank the alternatives. Such approach is often simply referred to as hybrid or an integrated approach in MCDM problems. In many of these hybrid approaches however, it is realized that in spite of the use of a hybrid method, a one-method approach could also realize the same ranking order. This has called into question the appropriateness of use of hybrid methods in MCDM.

This work first investigates the use of hybridized or integrated MCDM methods against one-method solutions to help determine (1) when a hybridized method solution is useful to a decision problem and (2) which MCDM methods are appropriate for setting criteria weights in a hybridized method and under what conditions. Further, based on the results in the first part of this work, the dissertation proposes a 2-tier hybrid decision making model using Conjoint Analysis and Intuitionistic Fuzzy - Technique for Order Preference by Similarity to Ideal Solution (IF-TOPSIS) method. The proposed hybrid method is useful in special cases of incorporating or merging preference data (decisions) of a large decision group such as customers or shareholders, into experts' decisions such as management board. The Conjoint analysis method is used to model preferences of the large group into criteria weights and subsequently, the Intuitionistic Fuzzy TOPSIS (IF-TOPSIS) method is used to prioritize and select competing alternatives with the help of expert knowledge.

Three numerical examples of such 2-tier (multi-level) decision model are provided involving (1) the selection of a new manager in a microfinance company where shareholder preference decisions are incorporated into board management decisions (2) an ideal company distributor selection problem where customer preferences are merged into management decisions to arrive at a composite decision and (3) selection of recruitment process outsourcing vendors where HR managers' preferences and trade-offs are incorporated into management decision. Finally the work tests the reliability of decisions arrived at in each of the studies by designing novel sensitivity analysis models to determine the congruent effect on the decisions.

Keywords: Multi-Criteria Decision Making (MCDM); Hybridized; Integrated; Criteria weights; Fuzzy sets; Intuitionistic fuzzy sets; Conjoint Analysis; TOPSIS.

ABSTRAKT

Vícekriteriální rozhodování (MCDM - multi-criteria decision making) s fuzzy nastavením má velmi rozsáhlé uplatnění v průmyslu a akademické obci. Současným trendem v těchto pracích není použití jedné, ale více metod MCDM v problémech ohodnocování pořadí a selekce. Například, při řešení problému výběru dodavatele může být použitý analytický hierarchický proces (AHP) nebo metoda Analytic Network Process (ANP) k nastavení vah kritérií a jiná metoda MCDM pro klasifikaci alternativ. Takový přístup je hodnocen jako hybridní nebo integrovaný přístup v problémech MCDM. Nicméně, v mnoha z těchto hybridních přístupů i přes použití metody hybridní by prostá jedna metoda mohla dosáhnout stejného ohodnocení pořadí. Je to pak otázkou vhodnosti použití hybridních metod MCDM.

Tato práce poprvé prozkoumává použití hybridních nebo integrovaných MCDM metod vůči řešení jednou metodou, aby pomohla určit, (1) kdy je řešení hybridní metodou užitečné pro rozhodovací problémy, (2) které MCDM metody jsou vhodné pro nastavení vah kritérií v hybridních metodách a za jakých podmínek. Dále, založeno na výsledcích v první části práce, disertační práce navrhuje 2-stupňový hybridní rozhodovací model využívající Conjoint Analysis (sdruženou analýzu) a Intuitionistic Fuzzy - Technique for Order Preference by Similarity to Ideal Solution (intuicionistickou fuzzy techniku pro preferenci pořadí podle podobnosti k ideálnímu řešení - IF-TOPSIS) metodou. Navržená hybridní metoda je užitečná ve speciálních případech zahrnující nebo slučující preferenční data (rozhodnutí) z velké skupiny lidí, jako jsou zákazníci nebo akcionáři do rozhodnutí expertů, např. vedení podniku. Conjoint analysis metoda je použita pro modelování preferencí velké skupiny do vah kritérií a následně, Intuitionistic Fuzzy TOPSIS (IF-TOPSIS) metoda je použita k upřednostňování a výběru protichůdných alternativ s pomocí znalostí expertů.

V práci jsou uvedeny tři numerické příklady takového 2-stupňového (multi-úrovňového) modelu a zahrnují: (1) výběr nového manažera ve společnosti, kde preferenční rozhodnutí akcionářů jsou zahrnuta do rozhodnutí vedení managementu, (2) problém výběru ideálního distributora společnosti, kde jsou zákaznické preference sloučeny do rozhodnutí managementu k dosažení složeného rozhodnutí a (3) výběr z náborového procesu outsourcingové společnosti, kde manažerské preference a kompromisní řešení jsou zahrnuta do rozhodnutí managementu. Posledním bodem práce je testování spolehlivosti rozhodnutí v každé studii pomocí modelu citlivostní analýzy, aby se určil souhlasný efekt na rozhodnutí.

Klíčová slova: Vícekriteriální rozhodnutí (Multi-Criteria Decision Making - MCDM); Hybridní; Integrovaný; Váhy kritérií; Fuzzy množiny; Intuicionistické fuzzy množiny; Sdružená analýza (Conjoint Analysis); TOPSIS.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS AND ABBREVIATIONS

IFS	Intuitionistic fuzzy sets
MCDM	Multi-Criteria Decision Making
F-MCDM	Fuzzy Multi-Criteria Decision Making
TFN	Triangular Fuzzy Number
IFN	Intuitionistic fuzzy numbers
IFWA	Intuitionistic Fuzzy Weighted Averaging Operator
IFHWA	Intuitionistic Fuzzy Hybrid Weighted Averaging
OWA	Ordered weight aggregation
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
DEMATEL	Decision Making Trial and Evaluation Laboratory
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
SIR	Superiority and inferiority ranking method (SIR method)
ELECTRE	ELimination and Choice Expressing REality
WPM	Weighted product model
WSM	Weighted sum model
DEA	Data Envelopment Analysis
VIKOR	Multi-criteria Optimization and Compromise Solution
PROMETHEE	Preference Ranking Organization METHod for Enrichment of Evaluations
MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique
MAUT	Multi-attribute utility theory (MAUT)
UTADIS	Utilities Additives DIScriminantes
GAIA	Geometrical Analysis for Interactive decision Aid
DM	Decision Makers
SWARA	Step-wise Weight Assessment Ratio Analysis
CBCA	Choice-Based Conjoint Analysis
CA	Conjoint Analysis
TCA	Traditional Conjoint Analysis

ACA	Adaptive Conjoint Analysis
MONANOVA	Monotone Analysis of Variance
IF-TOPSIS	Intuitionistic Fuzzy Technique for Order Preference by Similarity to Ideal Solution
PAPRIKA	Potentially All Pairwise Rankings of all Possible Alternatives
MODM	Multi-Objective Decision Making
MADM	Multi-Attribute Decision Making
F-MCDM	Fuzzy Multi-Criteria Decision Making
IFPIS	Intuitionistic Fuzzy Positive Ideal Solution
IFNIS	Intuitionistic Fuzzy Negative Ideal Solution
DANP	DEMATEL-based ANP
PCA	Principal Component Analysis
XLSTAT	Statistics and Data Analysis Software in Excel

1 INTRODUCTION

Decision making is largely considered a natural process for all beings with cognitive abilities. It often arises when one decides to choose the best possible option among a range of alternatives. When a decision problem involves only a single criterion, the approach to decision making is often intuitive and much easier to make. This is because the alternative (option) that receives the highest positive ratings is adjudged the 'best' or the most ideal among the sets of alternatives [1]. In real-life situations however, the nature and complexities of decision problems require robust methodological approaches that do not only consider multiple criteria to arrive at optimal decisions, but also the dependencies and confliction among the criteria. Multi-Criteria Decision Making (MCDM), a sub-discipline of operations research, surged in prominence when decision makers realized that, with a growing complexity in problems, it was unrealistic to proffer solutions using only one criteria or objective function [55]. In view of this, the methodological approaches adopted in Multi-Criteria Decision Making (MCDM), depend largely on the nature and level of complexity of the underlying problem. In practice, the MCDM approach is often either one of deterministic, stochastic, fuzzy or combinations of any of the above methods. It has been observed and widely accepted that (MCDM) problems tend to grow much more complex when the underlying information is uncertain, subjective or imprecise [1]. For instance, finding the value of the j-th alternative in a supplier selection problem when 'quality of product', a subjective criterion, is used to judge the alternatives [2]. In such situations, linguistic statements are deemed appropriate to describe the performance of the alternatives with respect to the criteria considered. When this approach is followed, quantifying such linguistic statements using deterministic MCDM approaches can be difficult and often inappropriate. One of the widely dependable and effective theories used to model linguistic human decisions and judgements is the concept of *fuzzy set theory*.

Finding suitable methodologies to deal with uncertain or subjective information always presented huge challenge to researchers in the past [6], forcing many to adopt probability theory or present uncertainty as randomness. This challenge generated interests in research in mathematical theories aimed at adequately dealing with situations of uncertainty [6]. Consequently, some of the most popular theories that emerged for uncertainty modelling were fuzzy sets [3] and possibility theory [7] by Zadeh, the rough set theory by Pawlak [8], Dempster [9], [10] and Shafer's [11] evidence theories. As a result of the introduction of such theories, it became clear that uncertainty can present itself in many forms and therefore requires caution and scrutiny in finding the relevant methodology suitable for a particular problem [6]. For instance, in the area of fuzzy sets, many generalized variants of the theory have been proposed aimed at dealing with different kinds of uncertainty and thereby improving the science of uncertainty modelling. Some of these fuzzy generalized forms are the rough sets [58], intuitionistic fuzzy sets

[12, 13, 14], interval-valued fuzzy sets [59], hesitant fuzzy sets [60], soft sets [61], and type-2 fuzzy sets [62, 63] among others.

Since its introduction, fuzzy set approaches have been found a suitable tool in modelling human knowledge especially in decision making problems that involve multiple subjective criteria. Over the years, a range of decision support techniques, methods and approaches have been designed to provide assistances in human decision making processes [1]. Some of these methods are the Analytical Hierarchy Process (AHP) [45], Analytic Network Process (ANP) [46], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [47], VIseKriterijumska Optimizacija I Kompromisno Resenje, which in English is Multi-criteria Optimization and Compromise Solution (VIKOR) [48], Simple Additive Weighting Method (SAW) [49], ELimination Et Choice Translating REality (ELECTRE) [50], Preference Ranking Organization METHods for Enrichment Evaluations (PROMETHEE) [51], Decision Making Trial and Evaluation Laboratory (DEMATEL) [52] among several others [22, 23]. Though many of these methods and techniques were first proposed with a focus on quantitative or deterministic multi-criteria decision making, they have all since been extended to deal with situations of imprecision or uncertainty in data using fuzzy sets. Consequently, fuzzy versions of AHP, TOPSIS, PROMETHEE, VIKOR and many others have seen wide spread applications in many areas [5].

Besides the fuzzy extensions of the various MCDM methods, another recent trend in fuzzy MCDM literature, is the use of so-called hybridized methods that combine more than one of existing MCDM methods in ranking and selection decision problems. In such instances of combining existing MCDM methods to solve decision problems, words such as ‘hybrid’ and ‘integrated’ are often used to describe the adopted methodological approach. An example could be a hybrid composed of AHP-TOPSIS or ANP-VIKOR etc. The premise for such combinations or hybrid methods is to adopt different MCDM methods to tackle different stages in a typical structured multi-criteria decision making. In typical MCDM solution, there are a number of processes and steps that are often followed. Some of these are identifying the problem, constructing the preferences, weighting the criteria and decision makers, evaluating the alternatives, and determining the best alternatives [14, 15, 16]. Figure 1 shows a flowchart of some of the processes involved in typical hybrid MCDM solutions. In such hybrid and integrated MCDM methods, the practice is to adopt different MCDM methods to set criteria weights, aggregate decision makers’ preferences or rank the alternatives. Since each MCDM method has its strengths and weaknesses, Decision Analysts must ensure the appropriateness of a hybrid method to a particular decision problem. Another issue with the hybrid methods is that, in most cases when a single MCDM method is utilized for the same underlying problem, the output as far as the ranking of the alternatives tends to be the same.

This dissertation investigates the use of hybridized integrated MCDM methods against single-method solutions to help determine (1) the appropriateness and usefulness of hybridized methods to an MCDM problem and (2) which MCDM methods are considered appropriate for setting criteria weights or ranking alternatives in hybridized solutions and under what conditions.

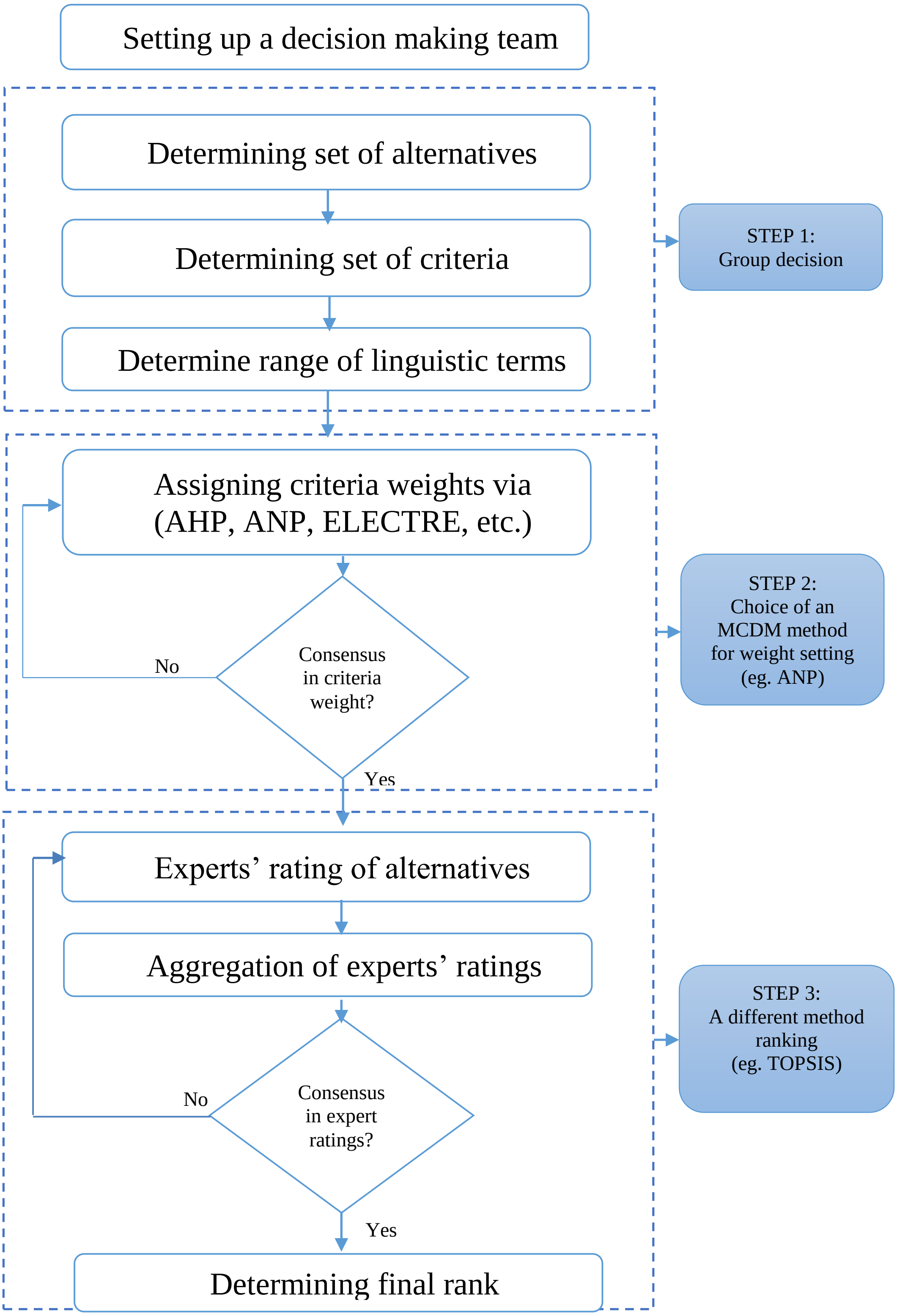


Fig. 1: Schematic diagram of a typical hybrid fuzzy MCDM Technique

Further, based on the results in the first part of this work, the dissertation proposes a novel 2-tier hybrid decision making model using Conjoint Analysis and intuitionistic fuzzy TOPSIS method to demonstrate an ideal application of such hybrid method. The usefulness of the proposed hybrid method is demonstrated in special cases of incorporating or merging preference decisions of a large decision group (non-experts DMs) such as customers or shareholders into decisions by a relatively small group (experts), to form a single composite decision. The Conjoint analysis method is used to model preferences of the large group into criteria weights whiles the Intuitionistic Fuzzy - Technique for Order Preference by Similarity to Ideal Solution (IF-TOPSIS) is used to rank competing alternatives with the help of expert knowledge.

To demonstrate the applicability of the proposed hybrid methods, three numerical examples of such 2-tier (multi-level) decision making model are provided in case studies. The first case study focusses on the selection of a new manager in a microfinance company where shareholder preference decisions are incorporated into board management decisions. The second numerical example models the incorporation of customer preferences into management decisions involving the selection a company distributor. Finally the third study looks at the selection of recruitment process outsourcing vendor where Human Resource (HR) managers' preferences and trade-offs are incorporated into management decision. The work further tests the reliability of the 'ranking' decisions in each of the case studies by designing novel sensitivity analysis to determine the congruent effect on the decisions when certain input parameters are altered.

The dissertation is divided into 2 main parts in an 11 chapter series. The first part composes of the introduction, aims of the dissertation, state of the art, theoretical foundations of MCDM, fuzzy set theory, investigation into hybridized MCDM methods and the concept of weights setting in MCDM. The second part of the 4 remaining chapters, comprises of the proposed hybrid method, numerical examples, sensitivity analysis as well as conclusion and discussion. In the first part, a thorough introduction and review of hybridized fuzzy multi-criteria decision approaches are presented. This is followed by the aims of the dissertation and the research problem, the identified research space and an outline of how the dissertation fills the research space. Subsequently in the second part, the proposed hybrid method of Conjoint Analysis – Intuitionistic Fuzzy TOPSIS method together with their underlying mathematical expressions and how the model works, are presented. This is followed by three numerical examples in chapter 9 that demonstrate the ideal applicability of the proposed hybrid method and how it compares with other fuzzy hybridized MCDM methods. Chapter 10, provides novel sensitivity analysis approaches to the results of the numerical examples, demonstrating how several input parameters can be changed to observe the overall effect on the final ranking of the alternatives. Finally, Chapter 11 summarizes the dissertation by presenting contributions to knowledge, identifying limitations in the work as well as the direction for future work.

2 AIMS OF THE DISSERTATION

The overall aim of the research is to expand knowledge on fuzzy multi-criteria decision making methods (MCDM). In particular, the dissertation reviews and compares fuzzy hybrid MCDM methods against single-method fuzzy MCDM solutions. This is to bring to light the appropriate use of the two approaches. A hybrid MCDM method is further proposed with numerical examples to demonstrate how it can be used in special decision problems. The following is an outline of the aims of the dissertation:

- To investigate the use of hybridized or integrated MCDM methods against one-method solutions. To determine:
  - when a hybrid method solution is useful to a selection problem.
  - which MCDM method is ideal for setting criteria weights in a hybridized method and under what conditions.
- To design a new hybrid MCDM method that incorporates user (consumers, shareholder, etc.) preferences and expert decisions in an ideal decision making situation. The proposed method is composed of Conjoint Analysis – Intuitionistic Fuzzy TOPSIS and are specifically used in the following:
  - Conjoint Analysis for setting criteria weights
  - Intuitionistic Fuzzy TOPSIS for ranking competing alternatives
- To test the proposed hybrid fuzzy MCDM model with real-life numerical examples and provide sensitivity analysis schemas to test the reliability of decisions.

3 STATE OF THE ART

The early 70s and 80s, saw a gradual rise in the development of theories and methods relating to the concept of multi-criteria decision making (MCDM). Though the exact name had not been conceived then, several researchers provided many important contributions which paved the way for the many MCDM methods in existence today. According to [72], the notion of Goal programming was a strong contributory influence to the development of MCDM concepts since most of the earliest MCDM topics centered on optimization. Following Goal programming, another concept that generated huge interests in the 70s to help shape the field of MCDM was the development of vector optimization algorithms. The focus on vector-valued objective function capable of computing multiple objective programs with all non-dominated solutions spawn interests especially among [74, 75, 76, 77, 78, 79] as cited in [73]. It was later realized according to [73], that there was a challenge or limitation with the vector-valued function as a result of the size of the growing non-dominated solutions. This challenge generated interests in alternate interactive solutions notably by [80, 81, 82] which were a step closer to setting up the field of MCDM. In particular, works by Sawaragi, Nakayama and Tanino [65] provided an extensive mathematical foundation and insight into the operations of MCDM. Their mathematical foundations were preceded by Hwang and Masud [66] and later Hwang and Yoon [47] who brought clarity into how many of the MCDM methods work and are distinct from each other. Zeleny [67] also focused on methods and decision processes with emphasis on the philosophical aspects involving multi-criteria decision making in general. Others like Vincke [68] strengthened and provided divergent views on how MCDM methods are approached. Steuer [69] strengthened the area of linear MCDM problems with useful theoretical constructs especially on how to analyze different MCDM problems. These developments enumerated above, strengthened the resolve of more researchers in the area of multi-criteria decision making (MCDM).

The domain of research in multiple criteria decision making has since evolved rapidly with several MCDM methods formulated to aid decision making in very complex decision problems [53]. The nature and level of complexity of the problem, however determine the methodological approach adopted. The approach could either be one of deterministic, stochastic, fuzzy or combinations of any of the above methods. During the last half of the century, a multitude of such methods has been developed to adequately deal with different kinds of decisions problems. Some of the notable MCDM methods that have had wide spread use and which were developed as a result of the theoretical foundations laid earlier in the 60s and 70s are the following. The *Analytical Hierarchy Process* [45] which is one of the most widely used MCDM methods and its variant, the *Analytic Network Process* (ANP) [46] method, were both developed by Saaty. Another widely used method both in industry and academia is the *Technique for Order*

Preference by Similarity to Ideal Solution (TOPSIS) method propounded by Hwang and Yoon [47]. Others are Opricovic's *VIseKriterijumska Optimizacija I Kompromisno Resenje*, which in English is Multi-criteria Optimization and Compromise Solution (VIKOR) [48], *ELimination Et Choice Translating REality* (ELECTRE) by Roy [50] and subsequently by Roy and Vincke [53], *Preference Ranking Organization METHods for Enrichment Evaluations* (PROMETHEE) by Brans and Vincke, [51], *Decision Making Trial and Evaluation Laboratory* (DEMATEL) [52]. Some of the very recent MCDM methods additions are the *Measuring Attractiveness through a Category Based Evaluation Technique* (MACBETH) by Costa, Bana, and Vansnick [70], *Potentially all pairwise rankings of all possible alternatives* (PAPRIKA) by Hansen and Ombler [71], and *Superiority and inferiority ranking method* (SIR) by Xu [72].

Typically in decision analysis, there tends to be a structured series of processes such as: problem identification, preferences construction, alternatives evaluation and determination of best alternatives [1, 15, 16]. However, it must be noted that decision making is a laborious task that does not always start with problem identification and end with a choice of an alternative.

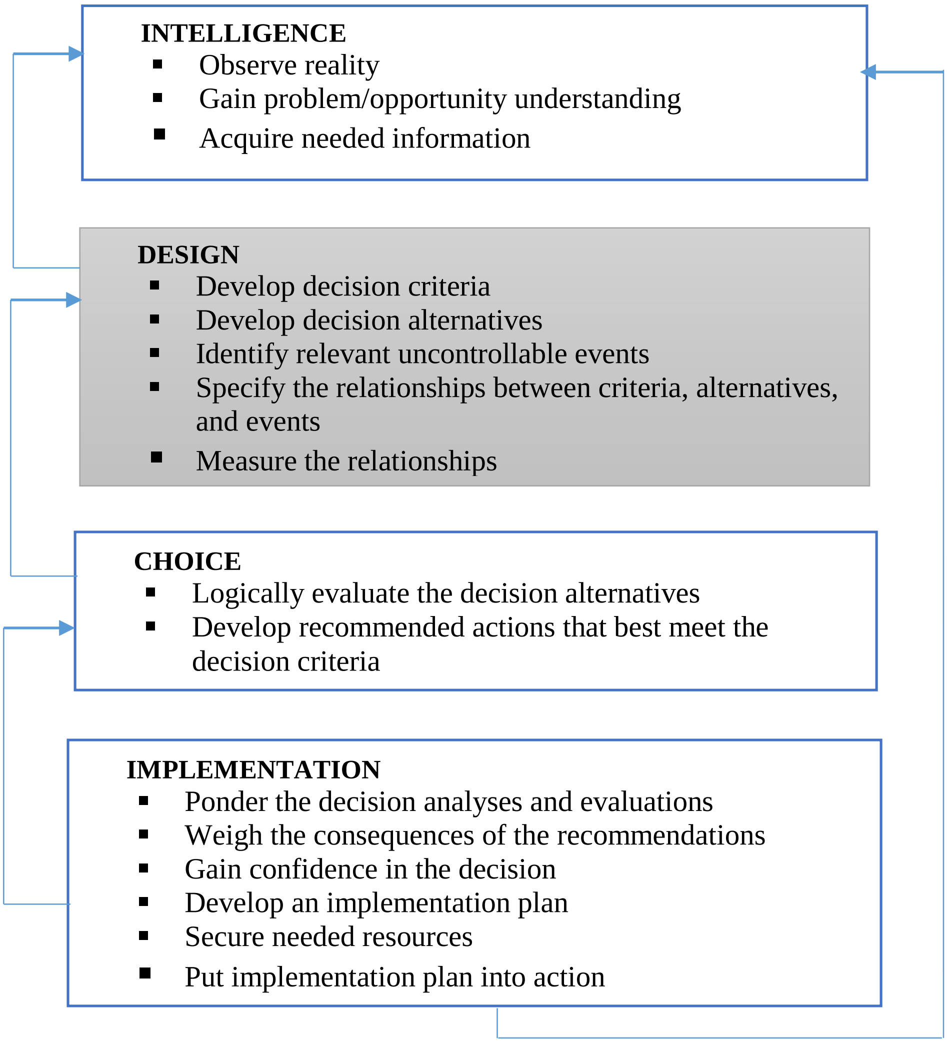


Fig. 2: Full decision making process (Source: [82])

Simon [82] proposed a paradigm that seemingly embodies the whole processes of human decision making. This widely used paradigm was first composed of three phases namely intelligence, design, and choice. A fourth phase, implementation, was later added as illustrated in figure 2. Simon describes the intelligence phase as when the decision maker 'observes the reality', gets an appreciation of the problem domain and seeks for opportunities out of the problem. Subsequently, the intelligence phase also gathers all relevant information about the problem to help arrive at an ideal solution. The next stage is termed the design phase where all relevant decision criteria, alternatives as well as events are modelled under a mathematical formulation. In MCDM problems, this is called the decision matrix. The paradigm also stresses that the relationships among the decisions, alternatives and relevant events are specified and measured [82] as cited in [83]. Finally, the implementation phase offers the decision maker the chance to ponder over the decision made and consider the consequences the decisions could potentially have over the circumstances. To carry out the implementation, it is also prudent that all the necessary resources needed are secured. When these are followed, then according to Simon's decision making paradigm, the implementation plan is ready to be executed. The decision paradigm must also be seen as a continuous loop where each stage in the process is constantly reviewed or evaluated especially as and when new information is received. In this dissertation, the focus is on stage 2, the design phase (as shaded in figure 2) where a new hybridized and integrated MCDM method is proposed for a special case of merging decision streams from two different sets of decision makers.

In the following section, the theoretical foundations of the concept of MCDM are mathematically explained. Key terminologies used in MCDM approaches are also outlined.

- **THEORETICAL PART**

4 THEORETICAL FOUNDATIONS OF MCDM

Multi-Criteria Decision Making (MCDM) is generally composed of two approaches; multi-objective decision making (MODM) and multi-attribute decision making (MADM). However the terms MADM and MCDM are most often used interchangeably to mean the same thing. In this dissertation, MCDM would most often be used to mean MADM. More generally, MCDM (MADM) thrives on the assumption that the underlying decision problem has a finite set of alternatives [55]. In such decision problems, the set of competing alternatives are therefore predetermined [2]. On the other hand, MODM works best in an environment where the decision space is continuous [2], meaning an infinite subset of a vector space defined by restrictions.

Formally, an MCDM problem

$$\rho = (A, f) \tag{1}$$

is referred to as multiple attribute decision making (MADM) problem if A is finite. In this case problem ρ can be expressed as an MADM decision matrix $S \in R^{u.v}$ where

$$A = \{a_1, a_2, ..., a_u\} \tag{2}$$

and

$$a_h = \{s_{h1}, s_{h2}, ..., s_{hv}\} \tag{3}$$

for all $h \in \{1, ...u\}$

An MCDM problem $\rho = (A, f)$ is referred to as multiple objective decision making (MODM) problem if A can be written as:

$$A \subseteq R^n, A = \{a \in R^n : g_i(a) \leq 0, i \in \{1, ..., m\}\} \tag{4}$$

with restrictions

$$g : R^n \to R, i \in \{1, ..., m\}. \tag{5}$$

In MCDM or more appropriately, MADM problems, a number of terminologies have become industry standards guiding both research and industry applications. Some of these terminologies are the following.

Decision Maker (DM)

In MCDM problems, the decision maker typically initiates and ends the decision process. The DM in this regard is responsible for structuring the decision problem, determining the sets of alternatives, choosing an alternative and finally reviewing the decisions made. In practice however, DMs are mostly experts with considerable knowledge regarding both the alternatives and the criteria to be used in judgement [1]. However, the DM is most always not a decision analyst. The decision analyst is one who aids the decision making process by offering appropriate formal methods (MCDM methods) to guide or assist decision makers. Therefore the decision maker (who gives judgements) and the decision analysts who chooses appropriate methods suitable for the problem at hand, are all very important to the process.

Alternatives

In structured decision making, alternatives usually refer to the choices of action or options available to the decision maker. These competing alternatives are pre-screened, prioritized and finally the best is/are selected [2]. In this dissertation, the focus is on MADM problems and therefore alternatives are considered to be finite.

Criteria

The term criteria is used interchangeably with the terms attributes and goals, to describe the different performance measures from which the alternatives are assessed. In decision problems where multiple criteria are considered, the practice is to arrange the criteria in a hierarchical structure especially when they are so much in number. In this case, major criteria are created with associated sub-criteria. In rare cases, some sub-criteria may also have their related sub sub-criteria. Furthermore, it is observed that in some cases, the multiple criteria tend to conflict with one another especially when there are tradeoffs among two or more criteria. An example is criteria; cost and quality, when the goal is to minimize cost and concurrently maximize quality [2]. However, regarding conflicts in MCDM, Zeleny [64] argues that the conflicts arise not among criteria but rather among the alternatives.

Decision Weights

In MCDM, most of the methods employ the notion of criteria weights where the range of criteria used in judgements are prioritized in terms of their relative or contributory importance to the final decision. In other instances, not only are the criteria weighted, the decision makers are also sometimes assigned weights on the assumption that not all the DMs are equal in importance [1,2, 55]. Typically in most MCDM methods, the weights are normalized so that their aggregation adds up to one. There are a number of ways of estimating the weights of criteria or DMs such as through various optimization approaches or by the use of different MCDM methods [2, 53]. Chapter 7 expounds the concept and development of weighting methods especially in MCDM.

Decision Matrix

In structured MCDM, the decision problems are formulated for easy analysis in a matrix format involving decision makers, the alternatives and the measuring criteria. More formally, a decision matrix for DM k is an (m x n) matrix where $A = \{A_1, A_2, \dots, A_m\}$ are the set of alternatives to be considered, $C = \{C_1, C_2, \dots, C_n\}$, the set of criteria and, $k = \{D_1, D_2, \dots, D_d\}$ the sets of decision makers. Equation. (6), shows a decision matrix for decision maker, $k = 1,2, \dots, d$

$$k = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \end{array}, i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{6}$$

$$W = [w_1, w_2, \ldots, w_n] \quad , j=1,2,\ldots,n \tag{7}$$

where x_{ij} is the rating of alternative A_i with respect to criterion C_j. Similarly, it also assumes that there is a predetermined weight for the criteria indicating the relative importance of each criterion in relation to the decision making. In this regard, w_j in Eq. (7) denotes the weight of a criterion for j = 1, 2, 3, ... , n).

4.1 Taxonomy of MCDM Problems and Methods

Decades ago, decision makers may have felt helpless when faced with multi-criteria decision problems. Today, whiles the decision maker would not be overly lacking in terms of solutions to complex multi-criteria problems, the sheer numbers of different MCDM methods available, also poses a challenge to DMs in terms of the appropriateness of a method to a problem. In view of this, classifying MCDM methods under various groupings with similarities in features, is seen as a step to minimizing problems with method choice abuses. Classifying MCDM problems and its range of solutions and methods, help to tailor appropriate methods and solutions for specific problems. Further, it is also useful since every MCDM method has its own sets of characteristics unique to how it approaches decision problems. It must also be stated that none of the MCDM methods is absolutely perfect nor can offer solutions to all decision problems.

Multi-criteria decision making methods may be classified in several ways. For example the distinction can be made according to (1) the problems suitable for the method (2) number of DMs involved (3) the nature of the alternatives (4) the kinds of data used by the methods and (5) the solution appropriateness to the problems [2]. When the classification is based on the number of decision makers, the outcome is either one of a group decision making or a single person decision making. On the other hand, when the consideration is on the kind of data to input into the method, then we may have deterministic, stochastic, or fuzzy MCDM methods [2]. According to [84], MCDM methods may also be grouped under two main classes; continuous and discrete methods, when the nature of the alternatives involved is considered. The continuous methods belong to the class of multi-objective decision making (MODM) problems where the focus is finding an optimal quantity that can be varied infinitely in a decision problem with an infinite subset of a vector space defined by restrictions. Typical MODM methods suitable for such problem scenarios are Goal programming and linear programming. Discrete MCDM methods on the other hand come under the MADM branch of MCDM where there is a pre-condition that a finite number of alternatives are considered. Because of this pre-condition, the set of alternatives are always predetermined. Further according to [85], MADM (discrete) methods are also grouped into ranking methods and weighting methods, which are further subdivided into qualitative, quantitative, and mixed methods [85]. For quantitative methods, the required data has to be in either cardinal or ratio data format [84]. Qualitative methods on the other hand require ordinal data.

Another useful form of classification is value and utility-based methods which aid decision-makers to effectively construct preferences. In particular is the Analytical Hierarchy Process which is by far the most popular and widely used method under the value and utility based approaches. Other notable ones are the Multi-attribute value theory (MAVT) and Multi-attribute utility theory (MAUT). Though the AHP and MAVT almost employ the same decision design paradigm, the AHPs approach in terms of setting criteria weights and rating alternatives differ considerably from MAVT approach. Some MCDM classifications also centre on certainty and uncertainty methods. The MAVT methods fall under the category of quantitative but riskless category whiles the MAUT as well as the 'French School' methods such as ELECTRE (Elimination and (Et) Choice Translating Reality) belong to the quantitative but risk category [84].

In real world applications, MCDM problems come with imperfect knowledge, vagueness or subjectivity mostly as a result of human judgements. This makes such decision problems complex to model. In view of this, information used by MCDM methods are also sometimes classified as either crisp or fuzzy. Information is considered crisp when it is deterministic or precise. On the other hand, an MCDM information is considered fuzzy when it is imprecise, subjective, incomplete or vague. Fuzzy set theory is used in dealing with this kind of information. The fuzzy data modelling extends the usual classification of MADM/MODM in MCDM to FMADM/FMODM (Fuzzy MADM/ Fuzzy MODM) [4].

4.2 MCDM Problem-Based Classification

Sufficient research indicates that multi-criteria decision problems appear in a wide range of areas and disciplines most notably in Economics, Operations research, Information systems, Environmental management, Logistics and supply chain management, Social Science among others [5]. However, irrespective of where the decision problems appear, they typically fall under four categories enumerated by Roy [53] and cited in [55]. These types of decision problems are as follows:

I. *Choice problem*. In choice or selection decision problems, the objective is to choose or select a best option among a finite set of competing alternatives. A typical example is a car choice problem where the best car is selected from a range of options under some criteria.

II. *Sorting problem*. In sorting problems, alternatives are grouped into ordered and predefined groups that share similar characteristics or features. For example, in employee performance evaluations, they may be grouped into different classes according to how they are performing as in: ‘over-performing’, ‘fairly-performing’ and ‘under-performing’ employees’. Such sorting could help in administering reward and punitive measures. Sorting decision problems are also sometimes employed in initial screening to precede a selection problem.

III. *Ranking problem*. Decisions problems that require ranking solutions typically order alternatives from the best or the most ideal to the worst through pairwise comparisons or through distance based measures. For instance, in job positions that plan to hire more than one candidates, performances during the interview are ranked to select deserving candidates.

IV. *Elimination problem.* This acts as a branch of sorting decision problems where the focus is to eliminate unwanted options with comparable features or which do not meet a certain requirements from a host of options. In such scenarios, a minimum threshold value is set to eliminate options that do not meet the threshold.

Table 1. MCDM decision problems and their appropriate methods

MCDM Methods	**Choice/Selection Decision**	**Ranking Decision**	**Sorting Decision**	**Descriptive decision**
AHP	✓	✓	×	×
ANP	✓	✓	×	×
AHPSort	×	✓	×	×
DEA	✓	✓	×	×
DEMATEL	✓	✓	×	×
ELECTRE I	✓	×	×	×
ELECTRE III	×	✓	×	×
ELECTRE-Tri	×	×	✓	×
FlowSort	×	×	✓	×
GAIA	✓	✓	×	×
Goal Programming	×	×	×	✓
MACBETH	✓	✓	×	×
MAUT	✓	✓	×	×
PROMETHEE	✓	✓	×	×
TOPSIS	✓	✓	×	×
UTADIS	×	×	✓	×
VIKOR	✓	✓	×	×

In Table 1, popular decision making methods that are generally deemed appropriate for solving peculiar problems are presented. However, to solve these decision problems, a formal analysis approach is often necessary to understand the demands of the underlying problem. Three of such formal analysis methods as identified by [16, 57] and cited in [1] are the descriptive, prescriptive and normative formal analysis. The descriptive analysis approach focuses on problems that DMs actually provide solutions to whiles prescriptive analysis reviews and identifies methods appropriate for DMs to use to solve decisions problems. Normative analysis on the other hand addresses the kinds of problems that DMs should ideally solve. This dissertation combines the prescriptive and the normative formal analysis approach in both investigative and design approaches of hybrid MCDM methods.

Again in Table 1, it is shown that each method has its own strengths, weaknesses and a general limitation regarding the kinds of problems they can solve. According to [85], the great diversity of MCDA methods, though a positive development, also presents problems especially of choice. Whiles there has not been any framework that helps to decide which method is perfectly appropriate for a particular problem, Guitouni [86] proposed a preliminary investigative framework to aid in the difficult situation of choosing an appropriate multi-criteria method for a suitable problem. In the framework, Guitouni [86] explains the different ways of choosing an MCDA method specific to problems. In Tables 2 and 3, are the guides to the kinds of input and output information required as well as the computational effort involved. For example the framework explains that if the ‘utility function’ for all of the criteria are known, then the multi-attribute utility theory (MAUT) is appropriate. It must however be recognized that constructing such utility functions comparatively requires a greater effort. The concept of pairwise comparison can also be used to group some of the MCDM methods. In particular are AHP and MACBETH that support this approach of pairwise comparison of either or both the criteria and the alternatives. However, for AHP, pairwise comparison is based on a ratio scale whiles an interval scale pairwise evaluation is used for MACBETH [84, 86]. The challenge of choice between the AHP and MACBETH should therefore be decided based on how best each scale, whether ratio or interval is suited to the underlying problem. According to [84], yet another way to approach the classification is to look at the key parameters involved. Regarding this approach, the PROMETHEE and the ELECTRE methods which both belong to the French School methods, are also seen to operate slightly differently from each other. PROMETHEE supports indifference and preference thresholds. On the other hand, the ELECTRE method requires indifference, preference and veto thresholds [84]. Further in terms of focusing on key parameters to distinguish one MCDM method from the other, are the elicitation methods which assist to define these key parameters. Another key MCDM method widely used is the TOPSIS method which essentially functions on a so-called positive and negative ideal solutions. TOPSIS is ideal if a decision analyst wants to avoid the PROMETHEE and ELECTRE which look for key parameters. Another method similar to TOPSIS with a similar distance-based approach is the VIKOR method. The VIKOR method also introduces a so-called best and worst values to separate the alternatives in terms of their performances.

Yet another useful way of classifying MCDM methods is based on the concept of *full-aggregation methods*, *outranking methods* and *Goal, aspiration or reference level methods*. In full aggregation approaches, also known as complete ranking or the American School approach, the alternatives considered in the decision problem have a global score where all alternatives are evaluated and ranked from best to worst or sometimes equal ranking [56, 84]. Methods such as TOPSIS, AHP and VIKOR are classic examples of such methods. One advantage with full aggregation methods is that, if an alternative is rated with a bad score on one criterion, it could be compensated for by a good score on another criterion [84]. On the other hand, outranking methods, also known as the French school methods, are based on pairwise comparisons. This implies that alternatives are compared 'head-on' two at a time using their preference scores. The preference or outranking degree indicates how much better one alternative is than another [84]. In Goal, aspiration or reference level methods, a goal is defined based on each criterion, and then alternatives closest to the ideal set goal or reference level are selected. Furthermore, with full aggregation methods, where the global score is the ultimate focus, it is sometimes possible to have incomparable alternatives especially where two alternatives have different profiles. For example, one alternative may be evaluated as 'better' on one criteria and the other alternative 'better' when evaluated on another set of criteria. According to [56], such incomparability is as a result of the non-compensatory nature of those methods. In view of this, [56] recommends that the type of output sought by a decision analyst should be given the same importance as the input data required. In Tables 2 and 3 are guides to understanding the input and output required for some selected MCDM methods. Some methods that belong to the MCDA family but are rarely classified as such are, data envelopment analysis (DEA) and conjoint analysis. DEA is primarily used for performance evaluation or benchmarking of units. As a typical deterministic method, DEA requires crisp or precise data inputs rather than subjective inputs. However with time there have been several extensions of DEA into fuzzy environments such as in [87, 88]. Conjoint analysis is used to elicit consumer preferences where tradeoffs are typically made. This dissertation employs conjoint analysis, though a method outside the scope of MCDM, to demonstrate its unique strengths and similarities to MCDM methods.

Table 2. Required inputs for MCDA sorting methods (source [56]).

	Inputs	Effort input	MCDM Method	Output
SORTING PROBLEM	Utility function	HIGH	UTADIS	Classification with scoring
	Pairwise comparisons on a ratio scale	↑	AHPSort	Classification with scoring
	Indifference, preference and veto thresholds	↓	ELECTRE-TRI	Classification with pairwise outranking degrees
	Indifference and preference thresholds	LOW	FLOWSORT	Classification with pairwise outranking degrees and scores

Table 3. Required inputs for MCDA ranking or choice method (source [56]).

	Inputs	Effort input	MCDM Method	Output
RANKING/CHOICE PROBLEM	No subjective inputs required		DEA	Partial ranking with effectiveness score
	Positive and negative ideal solutions	VERY LOW	TOPSIS	Complete ranking with closeness score
	Best and worst values (Distance based separation measures)	↑	VIKOR	Complete ranking with compromise
	ideal option and constraints		Goal Programming	Feasible solution with deviation score
	Indifference and preference thresholds		PROMETHEE	Partial and complete ranking (pairwise preference degrees and scores)
	Indifference, preference and veto thresholds		ELECTRE	Partial and complete ranking (pairwise outranking degrees)
	Pairwise comparisons on a ratio scale		AHP	Complete ranking with scores
	Pairwise comparisons on an interval scale		MACBETH	Complete ranking with scores
	Pairwise comparisons on a ratio scale and interdependencies	↓	ANP	Complete ranking with scores
	Utility function		MAUT	Complete ranking with scores
		VERY HIGH		

4.3 Other Types of Classification

Multi-criteria decision making methods can also basically be grouped into compensatory and non-compensatory methods [89]. This distinction is premised on 'whether advantages of one attribute can be traded for disadvantages of another or not' [89]. A decision problem is classified as compensatory if trade-offs are permitted among the set of criteria or attributes. On the other hand, the non-compensatory methods do not allow trade-offs among criteria. To this end, compensatory approaches, according to Yoon and Hwang [47, 89] are cognitively and computationally challenging but however produces optimal and rational decisions. Non-compensatory methods are largely seen to be simple both in terms of computational efforts and cognitive demands because each criterion stands independent in the evaluation. This means an inferiority or superiority in a criterion cannot be compensated for or balanced with an inferiority or superiority from another criterion. With the rationale behind each MCDM method been different and unique, the task of identifying an appropriate method is equally essential to ensuring optimal solutions to decision problems. The following are some methods which fall under the category of compensatory/non-compensatory.

4.3.1 Compensatory Methods

The Compensatory methods which allow for trade-offs among criteria can also be divided into the following subgroups.

- *Scoring Methods:* In scoring methods, a score also known as utility is used to express preference of one alternative or sometimes criterion over another. Some of the popular MCDM methods in this category are Simple Additive Weighting method (SAW) and the Analytical Hierarchy Process (AHP). Scoring methods typically design a preference scale on a range of [0,1].
- *Compromising Methods:* MCDM methods in this category use distance based separation measures to choose a best alternative. A best alternative is considered as the one closest to the ideal solution and concurrently farthest from the anti-ideal solution. MCDM methods that employ this approach are the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and VIseKriterijumska Optimizacija I Kompromisno Resenje, which in English is Multi-criteria Optimization and Compromise Solution (VIKOR). TOPSIS uses the terms positive ideal and negative ideal solutions to respectively describe the distances closest and farthest from the ideal solution. VIKOR on the other hand uses 'best' and 'worst' values to describe the distance separations [4].
- *Concordance Methods:* Concordance MCDM methods create a preference ranking in accordance with a given concordance measure. The alternative with relatively many highly rated criteria is chosen the best [72, 47]. The Linear Assignment Method is a popular method in this category.

4.3.2 Non-Compensatory Methods

MCDM methods that are non-compensatory and therefore do not allow for trade-offs among sets of criteria can also further be broken down into the following:

- *Dominance method*: In the dominated scenarios, all dominated alternatives are removed [72].
- *Maxmin method*: The maximin strategy is often described as conservative in that it basically identifies the worst (minimum) criteria value of each alternative and then selects the alternative that with the best (maximum) criteria value. It must be noted that the maximin method is only applicable when criteria (attributes) values are comparable [72, 86].
- *Maximax Method*: The maximax approach is a direct contrast to the maxmin method in that the best criteria value of each alternative are identified and the alternative with the maximum of these criteria values is the designated as the best alternative [72,86].
- *Conjunctive constraint method*: In this method, a minimum threshold value is set for each criterion in the selection process. By comparing how each criterion fares against the threshold value, the decision maker decides whether the standard meets his/her expectations. If they meet his expectations, then the DM has a satisfying alternative.
- *Disjunctive constraint method*: In this method, an alternative's best criterion or attribute value is the focus. The rest of the alternative's weak criteria are not factored in the evaluation [72].

4.4 MCDM Solutions

MCDM methods produce several kinds of solutions. These solutions are mainly based on a number of things most especially the nature of the solution. According to [47], since there are no absolute perfect solutions, MCDM problems may not always have a perfect outcome or solution. Some of the names given to different MCDM solutions are the following as explained in [72].

- *Ideal solution*: Typically in MCDM, an ideal solution is described as one that concurrently maximizes the benefit (profit) criteria and minimizes the cost criteria. Criteria that give some benefits or result in profits are maximized whiles those that bring some element of cost are as much as possible, minimized. In practice according to [47], ideal solutions are hard to come by and therefore decision analysts look next to non-dominated solutions.

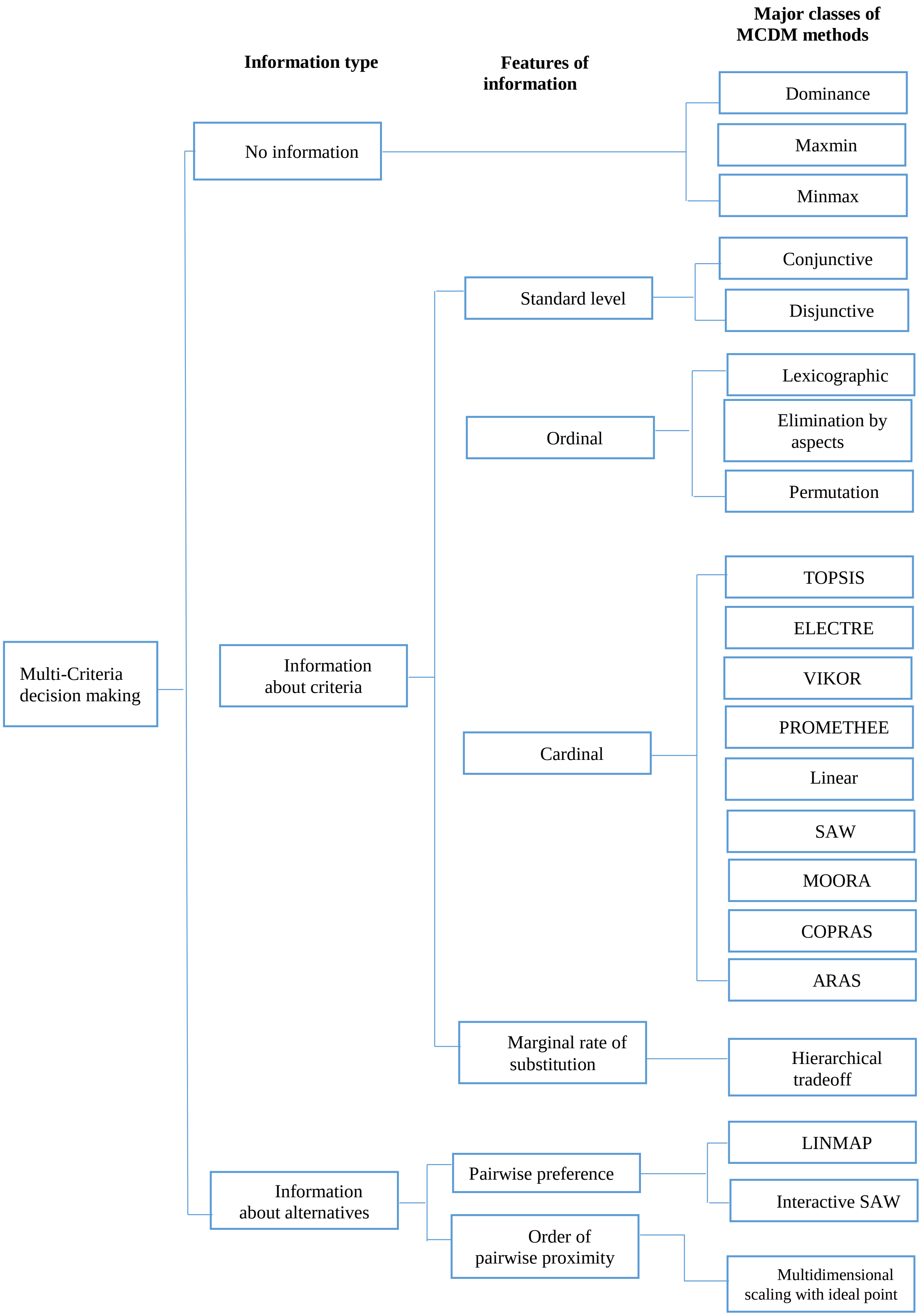


Fig. 3: Hierarchical outline of MCDM methods (source: [2])

- *Non-dominated solutions:* In MCDM problems, a non-dominated solution is one that relatively outperforms the other competing alternatives on all or most of the criteria under consideration. Dominated solution, which is not preferred, is one that is outperformed by all other alternatives in the evaluation.
- *Satisfying solutions*: A solution is described as satisfying if it meets most of the expectations of the decision analyst. Whiles a satisfying solution may not always be a non-dominated one, it is termed as the 'best' solution in the moment considering all the constraints.
- *Preferred solutions:* A preferred solution is a non-dominated solution that best satisfies the decision maker's expectations.

In figure 3, the various methods, approaches and solutions are outlined in a hierarchical structure to demonstrate where each method or solution belongs and by extension the appropriateness of a method to a solution.

Having reviewed MCDM problems, solutions, methods and their appropriateness to decision making, the following section looks at fuzzy logic – the tool that extends deterministic MCDM methods to deal with issues of uncertainty, imprecision and subjectivity in human decision making. Theoretical foundations of the fuzzy set theory are presented and further extended into its generalized form, the intuitionistic fuzzy sets.

5 FUZZY SET THEORY

The world is filled with several kinds of uncertainty which is often as a result of the absence of information, inaccuracies in measurements and a general imperfection in the information we receive. One source of such imperfect information is the use of our natural language to describe, communicate or share information [90]. Since our natural languages are imbibed with concepts and terms that do not have exact meaning or sharp boundaries, we often have disagreements in what a term or an idea exactly means. This is referred to as subjectivity in natural language usage. For example, human understanding of what the terms or phrases, *thin*, *large*, *much younger*, *quite beautiful*, *very secure*, *very warm*, *hot* etc. are relative in terms of the degree of definitional acceptance. These lack of clarity or imprecision in our expression of concepts is referred to as fuzziness or grey concepts. In the early 50s and 60s, several attempts were made at efficiently modelling such fuzzy concepts. One theory that won wide acclaim or acceptance was fuzzy logic by Lofti Zadeh [3]. The fuzzy set theory has therefore become one of the de-facto standards for modelling linguistic expressions that hold uncertainty, subjectivity and data with no sharp boundaries in terms of their intended meaning. The concept which is based on many-valued logic of relative graded membership, is a generalization of the classical set theory [91, 92]. In classical sets, an element in a set either belongs or do not belong but in fuzzy sets, an element can belong to more than one set [17]. In summary, classical sets is described as bi-valent with sharp crisp boundaries whiles fuzzy sets are many-valued with loose boundaries [6].

Though Zadeh's concept of fuzzy logic was conceived as a mathematical tool to deal with issues of uncertainty, imprecision and vagueness with formalized methods, the attempt at many-valued logic began centuries and millennia ago with works by Aristotle (law of the excluded middle). Plato, Łukasiewicz (three-valued logic), Knuth (three-valued logic). However, it was Zadeh who ultimately introduced the concept of infinite-valued logic in his seminal work titled "fuzzy sets" and therefore fuzzy logic [90, 17, 91]. Fuzzy logic creates a so-called member functions over a range of real numbers from [0,1]. Fuzzy logic brought to the fore, the limitations with using conventional or classical approaches in knowledge representation especially problems that hold uncertainty data. It was realized that the first order logic and classical probability theory were not appropriate methodologies for modelling uncertainty in our 'commonsense knowledge' which are lexically imprecise in nature [91]. According to Zadeh, Klir and Yuan [91], any fuzzy logic system should possess the following essential characteristics. That in fuzzy logic:

- exact reasoning has a limiting case of approximate reasoning.
- everything is a matter of degree.
- knowledge is interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.

- Inference is viewed as a process of propagation of elastic constraints.
- Any logical system can be fuzzified.

In summary, Zadeh [92] posits that, for better performance and appropriateness of use, the two most important characteristics that must be looked out for prior to the use of fuzzy logic systems are:

- Fuzzy systems are suitable for uncertain or approximate reasoning, especially for systems with a mathematical model that is difficult to derive.
- Fuzzy logic allows decision making with estimated values under incomplete or uncertain information.

5.1 Mathematical Foundations of Fuzzy Set Theory

The fuzzy set theory is a generalization or a comprehensive form of the crisp set. Formally, the classical or crisp set is defined in the following:

Definition 1: Let X and A be a set and its subset respectively with $A \subseteq$ X. Then

$$\lambda_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases} \tag{8}$$

where $\lambda_A(x)$is referred to as the characteristic function [6] of set A in X. Therefore a classical or crisp set can be defined as:

$$A = \{< x, \lambda_A(x)\} > |x \in A \tag{9}$$

The classical crisp set in Eqn. (9) is generalized into fuzzy sets as explained below:

A fuzzy set A' of X is a set of ordered pairs$\{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \dots, (x_n, \mu_A(x_n))\}$, characterized by a membership function $\mu_{A'}(x)$ that maps each element x in X to a real number in the interval [0,1]. The function value $\mu_{A'}(x)$stands for the membership degree x in A'.

Definition 2: A fuzzy set A' in X=$\{x\}$ is defined by:

$$A' = \{< x, \mu_{A'}(x)\} > |x \in X \tag{10}$$

where $\mu_{A'}: X \to [0,1]$ denotes the membership function of the fuzzy set $A'$and further describes the degree of membership of x to fuzzy set A'

It must be added that, since fuzzy set is a generalization of the classical crisp set defined in the interval [0,1], full membership and full non-membership of x in A'occurs when $\mu_{A'}(x) = 1$ and $\mu_{A'}(x) = 0$ respectively. This implies that every crisp set is in itself a fuzzy set.

If $X = \{x_1, x_2, \dots, x_n\}$ is a finite set and A' is a fuzzy set in X , a notation often used is,

$$A' = {}^{\mu_1}/_{x_1} + \cdots + {}^{\mu_n}/_{x_n} \tag{11}$$

where the term ${}^{\mu_i}/_{x_i}$, $i = 1, \dots, n$ denotes μ_i as the grade of membership of x_i in A' and the + sign also denotes the union.

Example 1:

Consider the temperature of a patient in degrees Celsius. Let $X = \{36.5, 37, 37.5, 38, 38.5, 39, 39.5\}$. The fuzzy set $A =$ "High temperature" may be defined

$$A' = \{< \mu_{A'}(x)/x\} > |x \in X$$

=0/36.5 + 0/37 + 0.1/37.5 + 0.5/38 + 0.8/38.5 + 1/39 + 1/39.5, where the numbers 0, 0.1, 0.5, 0.8, and 1 express the degree to which the corresponding temperature is high.

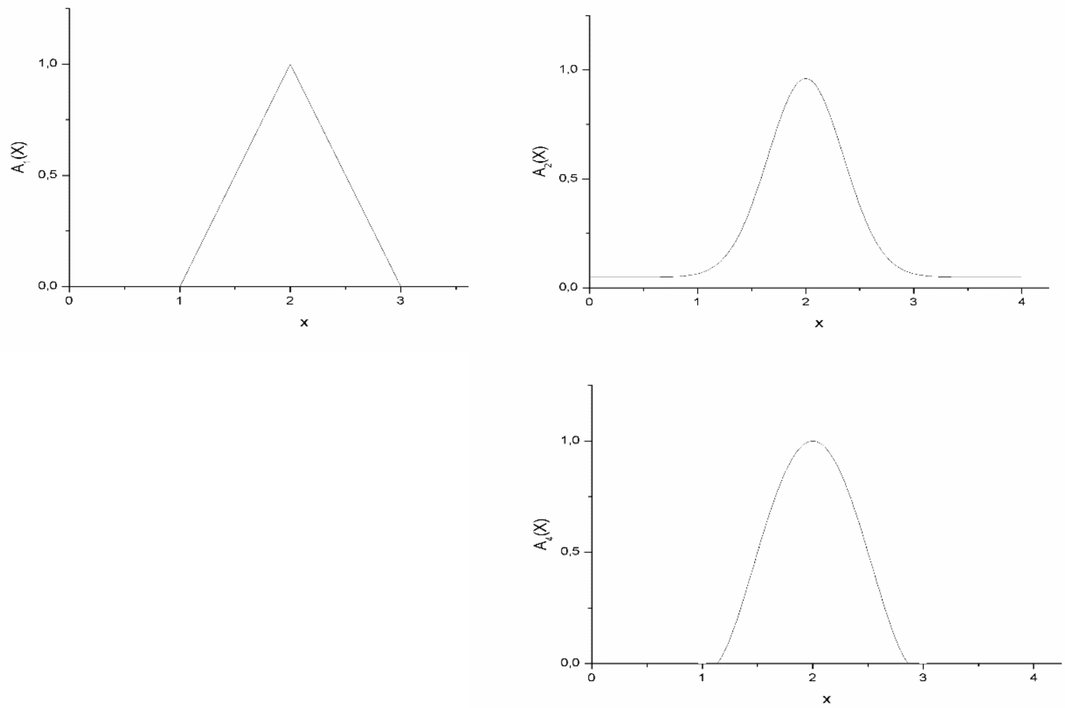


Fig. 4: Examples of membership functions that may be used in different contexts to characterize fuzzy sets.

The 3 fuzzy sets in figure 4 are similar in the sense that the following properties are possessed by each $A_i (i \in \mathbb{N}_4)$:

$$(i) A_i(2) = 1 \; and \; A_i(x) < 1 \; for \; all \; x \neq 2;$$

$$(ii) A_i \; is \; symmetric \; with \; respect \; to \; x = 2, that \; is \; A_i(2 + x) = A_i(2 - x) for \; all \; x \in \mathbb{R};$$

$$(iii) A_i(x) decreases \; monotonically \; from \; 1 \; to \; 0 \; with \; increasing \; difference \; of \; |2 - x|$$

Fuzzy sets assume certain properties such as ”increasing”, ”decreasing” or ”convex”, based on the properties of a membership function that characterizes the fuzzy set [92]. The fuzzy set property, 'increasing', is used to express such linguistic concepts as "long", "warm", "old" etc., on universes of length, temperature, age etc. Similarly, the fuzzy property of 'decreasing' could also be used to describe linguistic concepts of decreasing concepts in these universes would be "short", "cold", "young". In following section, more properties of fuzzy sets are introduced.

5.1.1 Properties of Fuzzy Sets

Definition 3: (support) Let A' be a fuzzy subset of X; the support of A', denoted *supp*(A'), is the crisp subset of X whose elements all have nonzero membership grades in A'.

$$supp(A') = \{x \in X \mid \mu_{A'}(x) > 0\}. \tag{12}$$

Definition 4. (normal fuzzy set) A fuzzy subset A' of a classical set X is referred to as '*normal*' if there exists an $x \in X$ such that $A' = 1$. Otherwise A' is subnormal.

Definition 5. (α-cut) An α-level set of a fuzzy set *A* of *X* is a non-fuzzy set denoted by $[A']^{\alpha}$ and is defined by

$$[A']^{\alpha} = \begin{cases} \{t \in X \mid A'(t) \geq \alpha\} & if\ \alpha > 0 \\ supp(A') & if\ \alpha = 0 \end{cases} \tag{13}$$

where (suppA) denotes the closure of the support of A'.

Example 2:

Assume X = {−2,−1, 0, 1, 2, 3, 4} and $A' = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$, in this case,

$$[A']^{\alpha} = \begin{cases} \{-1, 0, 1, 2, 3\} & if\ 0 \leq \alpha \leq 0.3 \\ \{0, 1, 2\} & if\ 0.3 \leq \alpha \leq 0.6 \\ \{1\} & if\ 0.6 \leq \alpha \leq 1 \end{cases}$$

Definition 6. (convex fuzzy set) A fuzzy set A' of X is called convex if $[A']^{\alpha}$ is a convex subset of X $\forall \alpha \in [0, 1]$.

Some Fuzzy Sets Algebraic Operations

Union: The union of two fuzzy sets A' and B' with membership functions $\mu_{A'}$ and $\mu_{B'}$ respectively is defined as the maximum of the two membership functions of A' and B'. This is referred to as the maximum criterion as expressed in Eqn. (14).

$$\mu_{A' \cup B'} = max(\mu_{A'}, \mu_{B'}) \tag{14}$$

Intersection: The intersection of two fuzzy sets A' and B' with membership functions $\mu_{A'}$ and $\mu_{B'}$ respectively is defined as the minimum of the two membership functions of A′ and B′. This is referred to as the minimum criterion as expressed in Eqn. (15).

$$\mu_{A' \cap B'} = min(\mu_{A'}, \mu_{B'}) \tag{15}$$

Complement: The complement of a fuzzy set A′ with membership function $\mu_{A'}$ is described as the negation of the stated membership function as expressed in Eqn. (16). This is known as the negation criterion.

$$\mu_{\overline{A}} = 1 - \mu_{A'} \tag{16}$$

5.1.2 Fuzzy Number

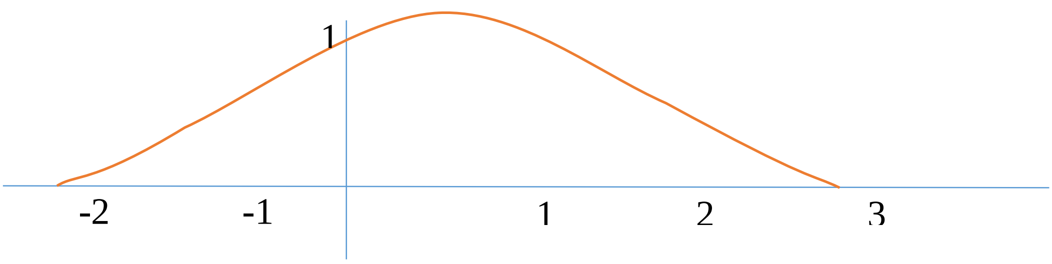
In fuzzy MCDM, a special case of fuzzy set known simply as a fuzzy number is introduced to capture vagueness and variations in the subjective ratings of an expert. A fuzzy number is a generalization of a real number ℝ where each single value is connected to a set of possible values assigned weights between 0 and 1 [18]. Fuzzy numbers use the [0,1] interval to rate a set of possible values by capturing the subjectivity, vagueness and imprecision of a subjective rating. Formally, a fuzzy number is defined as in definition 7.

Definition 7. (fuzzy number) A fuzzy number A′ is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support.

The notable types of fuzzy numbers are the trapezoidal and triangular fuzzy numbers. However, in most fuzzy multi-criteria decision making (MCDM) approaches, the Triangular Fuzzy Number (TFN) is preferred to trapezoidal because of its simplicity [19]. In the following, an illustration of some of the types of fuzzy numbers are briefly discussed.

*Definition 8***.** (quasi fuzzy number) A quasi fuzzy number A′ is a fuzzy set of the real line with a normal, fuzzy convex and continuous membership function satisfying the limit conditions

$$\lim_{t \to \infty} A'(t) = 0, \quad \lim_{t \to -\infty} A'(t) = 0.$$



Let A be a fuzzy number. Then $[A']^{\gamma}$ is a closed convex (compact) subset of ℝ for all $\gamma \in [0, 1]$. The following notations are introduced

$$a_1(\gamma) = \min[A']^{\gamma}, \; a_2(\gamma) = \max[A']^{\gamma}$$

In other words, $a_1(\gamma)$ denotes the left-hand side and $a_2(\gamma)$ denotes the right-hand side of the γ-cut. In view of this, it can be seen that

$$\text{If } \alpha \leq \beta \text{ then } [A']^{\alpha} \supset [A']^{\beta}$$

Definition 9. (triangular fuzzy number) A fuzzy set A′ is called triangular fuzzy number with membership function $y = f(x)$ expressed as a triplet (a, b, c) having the form presented in Fig. 5.

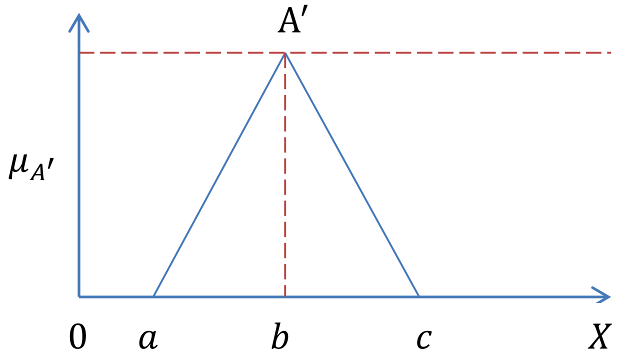


Fig. 5: Membership function of triangular fuzzy number

The equation of the straight line defined by the points (a,0) and $A(b,1)$ is given by $\frac{1}{b-a} = \frac{y}{x-a}$ or $y = \frac{x-a}{b-a}$. Similarly, the straight line defined by the points $A(b,1)$ and (c,0) is given by $\frac{1}{c-b} = \frac{y}{c-x}$ or $y = \frac{c-x}{c-b}$. Therefore the membership function $\mu_{A'}$ of the triangular fuzzy number (TFN) is defined as:

$$\mu_{A'}(x) = \begin{cases} 0 & x < a, x > b \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \end{cases} \tag{17}$$

In Figure 5, the value of x at b gives the maximal value of $\mu_{A'}(x)$, that is $\mu_{A'}(x) = 1$. The value x at a represents the minimal grade of $\mu_{A'}(x)$, i.e $\mu_{A'}(x) = 0$. The constants a and c represent the lower and upper bounds of the available area data respectively and reflect the fuzziness of the data under evaluation. In Figure 6, two triangular fuzzy numbers are presented to demonstrate the arithmetic operations underlying TFNs as seen in Eqs. (18) to (21) below.

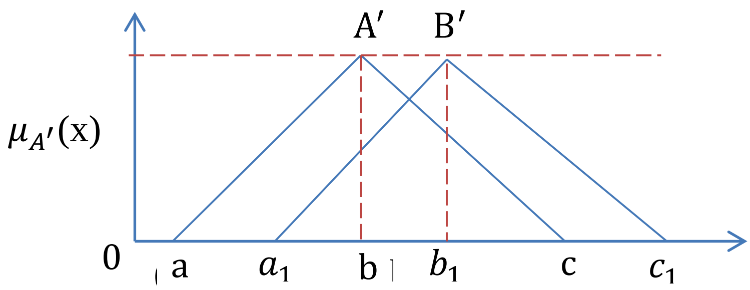


Fig. 6: Two triangular fuzzy numbers

Definition 9.1: Basic Triangular Fuzzy Number Operations

Let $A = (a, b, c)$ and $B = (a_1, b_1, c_1)$ be two triangular fuzzy numbers. Some basic operations on these two fuzzy triangular numbers are as follows:

$$A + B = (a, b, c) + (a_1, b_1, c_1) = (a + a_{1,}\ b + b_1, c + c_1) \quad (18)$$
$$A - B = (a, b, c) - (a_1, b_1, c_1) = (a - c_{1,}\ b - b_1, c - a_1) \quad (19)$$
$$A \times B = (a, b, c) \times (a_1, b_1, c_1) = (aa_{1,}\ bb_1, cc_1) \quad (20)$$
$$A \div B = (a, b, c) \div (a_1, b_1, c_1) = \left(\frac{a}{c_1}, \frac{b}{b_1}, \frac{c}{a_1}\right) \quad (21)$$

Definition 10. (trapezoidal fuzzy number) A fuzzy set A' is called trapezoidal fuzzy number with membership function $\mathrm{y} = \mathrm{f}(\mathrm{x})$ expressed as a quadruple $(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d})$ having the form presented in Fig. 7. The membership function is expressed as in Eq. (22).

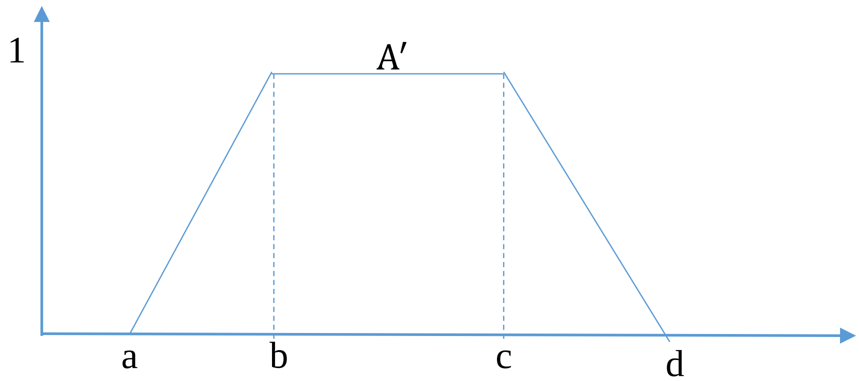


Fig. 7: Membership function of triangular fuzzy number

$$\mu_{A'}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b < x \leq c \\ \frac{d-x}{d-c}, & c \leq x < d \\ 0, & x > d \end{cases} \quad (22)$$

Definition 10.1: Basic Triangular Fuzzy Number Operations

In Eqns. (23) to (26), the arithmetic operations of two trapezoidal fuzzy numbers are presented. Let $A = (a, b, c, d)$ and $B = (a_1, b_1, c_1, d_1)$ be two trapezoidal fuzzy numbers. Some basic operations on these two fuzzy triangular numbers are as follows:

$$A + B = (a, b, c, d) + (a_1, b_1, c_1, d_1) = (a + a_{1,}\ b + b_1, c + c_1, d + d_1) \quad (23)$$
$$A - B = (a, b, c, d) - (a_1, b_1, c_1, d_1) = (a - d_{1,}\ b - c_1, c - b, d - a_1) \quad (24)$$
$$A \times B = (a, b, c, d) \times (a_1, b_1, c_1, d_1) = (aa_{1,}\ bb_1, cc_1, dd_1) \quad (25)$$
$$A \div B = (a, b, c, d) \div (a_1, b_1, c_1, d_1) = \left(\frac{a}{d_1}, \frac{b}{c_1}, \frac{c}{b_1}, \frac{d}{a_1}\right) \quad (26)$$

5.2 Generalized Forms of Fuzzy Sets

The discovery of different kinds of uncertainty, imprecision and vagueness in peculiar circumstances, brought about several extensions and generalizations to Zadeh's original fuzzy construct. These fuzzy extensions and generalizations are used in situations where the original fuzzy construct poses a limitation in terms of ability to efficiently solve a problem. The many fuzzy variants therefore also help in uncertainty modelling. Some of the widely accepted and used variants of the original fuzzy sets, are rough sets by Pawlak [8], intuitionistic fuzzy sets by Atanassov [12], interval-valued fuzzy sets by Grattan-Guinness [59], hesitant fuzzy sets by Torra [60], soft sets by Molodtsov [61] and type-2 fuzzy sets by [62, 63] among others. Though the fuzzy set theory has over the years, had wide ranging research interests, especially in control systems, pattern recognition, optimization and successfully been applied in diverse disciplines [93], its novel use in decision making seems to be one of the most popular. In decision making especially in the MCDM, the generalized forms of the fuzzy set theory have been tested and applied in several instances of modelling different forms of uncertainty in human decision making.

One fuzzy set generalization gradually becoming popular with MCDM practitioners is Atanassov's Intuitionistic Fuzzy Sets (IFS) [12]. Since its introduction, IFS has gradually become one of the most utilized fuzzy set generalizations after rough sets especially in MCDM problems [94]. Together, the two notions of uncertainty modelling; the classical (Zadeh) and intuitionistic (Atanassov) sets have been applied in many real-world MCDM applications spanning diverse disciplines. With a corresponding global interest in fuzzy MCDM methods and its generalized variants in both industry and academia, the dissertation demonstrates a combined use of intuitionistic fuzzy TOPSIS with conjoint analysis in a special case of merging decisions of different group decision makers.

In the following sections, the intuitionistic fuzzy set theory is briefly introduced. Furthermore, a comparison is made with both the classical and the fuzzy set theory to draw out the differences and similarities amongst them. In particular importance is how the three types of sets evolved from each other. The rest of the dissertation is organized as follows. Chapter 6 launches an investigation into the use of hybrid fuzzy MCDM methods against single method solutions. The result aids in the proposal of a new hybrid method.

5.3 Theory of Intuitionistic Fuzzy Sets

This section focuses on reviewing how existing theories evolved into other potent theories through so-called hybridizations and extensions. This is aimed at further development of a hybrid fuzzy MCDM method. To do this, an examination is made into how the classical set theory was generalized into fuzzy sets and subsequently extended or generalized into the intuitionistic fuzzy set theory. This implies that the classical set theory gave rise to the fuzzy set and its variant – the intuitionistic fuzzy set, all premised on offering theories that deal with different kinds of data. In the following sections, we compare the three sets approaches and their interrelationships. In Eqns. (8) and (9), the foundations of both the classical and the fuzzy sets theory were introduced. It was shown that fuzzy set is a generalized form of the classical crisp set defined in the interval [0,1] where full membership and full non-membership of x in A'occurs when $\mu_{A'}(x) = 1$ and $\mu_{A'}(x) = 0$ respectively. This implies that every crisp set is in itself a fuzzy set.

5.3.1 Intuitionistic Fuzzy Sets

In classical fuzzy set, a fuzzy set A in X characterized by membership functions is expressed as $A = \{\langle x, \mu_A(x)\rangle | x \in X\}$ where $\mu_A: X \rightarrow [0,1]$ describes the membership function of the fuzzy set A within the interval of [0, 1]. In intuitionistic fuzzy sets however, a set A in X is defined as:

$$A = \{\langle x, \mu_A(x), v_A(x) \ \rangle | x \in X\} \tag{27}$$

Where $\mu_A(x), v_A(x): X \rightarrow [0,1]$ respectively represent the membership and non-membership functions on condition that $0 \leq \mu_A(x) + v_A(x) \leq 1$. Additionally, IFS introduces a third construct $\pi_A(x)$, the intuitionistic fuzzy index which expresses whether or not x belongs to A.

$$\pi_A = 1 - \mu_A(x) - v_A(x) \tag{28}$$

The intuitionistic index in Eq. 28 measures the hesitancy degree of element x in A where it becomes obvious that $0 \leq \pi_A(x) \leq 1$ for each $x \in X$. A small value of $\pi_A(x)$ implies that information about x is more certain [20]. On the other hand, a higher value of the hesitancy degree $\pi_A(x)$ means the information that x holds is more uncertain. An intuitionistic fuzzy set can therefore fully be defined as

$$A = \{\langle x, \mu_A(x), v_A(x), \pi_A(x)\rangle | x \in X\} \tag{29}$$

Where $\mu_A \in [0,1]; v_A \in [0,1]; \pi_A \in [0,1]$

Definition 1. Basic arithmetic operations of intuitionistic fuzzy sets.

Let $A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), v_B(x)\rangle | x \in X\}$ be two intuitionistic fuzzy numbers. Some basic operations on these intuitionistic fuzzy numbers (IFNs) A and B applied in this research are expressed as follows: ("iff" means "if and only if"):

$$A = B \ \ iff \ \ (\forall\, x \in X)\big(\mu_A(x) = \mu_B(x) \ \ \& \, v_A(x) = v_B(x)\big) \tag{30}$$
$$A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), v_A(x) \cdot v_B(x)\rangle | x \in X\} \tag{31}$$
$$A \times B = \{\langle x, \mu_A(x) \cdot \mu_B(x), v_A(x) + v_B(x) - v_A(x) \cdot v_B(x)\rangle | x \in X\} \tag{32}$$

The complement A^c of a fuzzy set A in X corresponds to negation "not", and is defined as:

$$\mu_{A^c}(x) = 1 - \mu_A(x) \quad \text{for each } x \in X \tag{33}$$

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(v_A(x), v_B(x))\rangle | x \in X\} \tag{34}$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(v_A(x), v_B(x))\rangle | x \in X\} \tag{35}$$

Product of an intuitionistic fuzzy set and a real number is defined as in Eq. (36)

$$\lambda A = \left\{\langle 1 - (1 - \mu_A(x))^{\lambda}, (v_A(x))^{\lambda}\rangle x \in X\right\} \tag{37}$$

Power of IFS: $A^{\lambda} = \left\{\langle (v_A(x))^{\lambda}, 1 - (1 - \mu_A(x))^{\lambda}\rangle x \in X\right\}$ (38)

5.3.2 Summary of Comparison

In figure 8, a pictorial relationship among classical sets, fuzzy sets and intuitionistic fuzzy sets (IFS) is shown. This is further explained in figure 9 where the three sets are compared on a coordinate system to demonstrate how elements belonging to crisp sets, fuzzy sets and intuitionistic fuzzy sets interrelate [6]. In a crisp set, the coordinate system shows only two points to explain that in such sets, elements either belong fully or do not belong at all. It can further be shown that at points (μ =1 and ν =0) elements in a crisp set fully belong whiles at coordinates (μ = 0 and ν = 1), elements do not belong. In fuzzy sets, μ +ν = 1 demonstrates that the entire area (the middle part) connecting these points have elements belonging to a fuzzy set [6]. Finally in the case of intuitionistic fuzzy set, the strict condition: 0 ≤ μ +ν ≤ 1, allows both the segment with its ends and the interior of the shaded triangle at the bottom part of Figure 9c to represent the elements belonging to an intuitionistic fuzzy set. This means each of the lines inside the triangle depicts an image of the elements with the same value of intuitionistic fuzzy index. It can then be concluded that in the case of intuitionistic fuzzy sets, fuzzy sets and crisp sets are contained in it whiles fuzzy sets also respectively contain crisp sets as shown in figure 8 below [6].

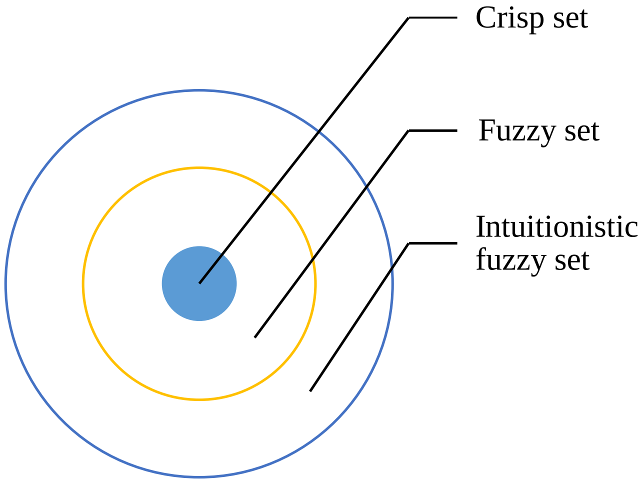


Fig. 8: Relationship among classical sets, fuzzy sets and intuitionistic fuzzy sets

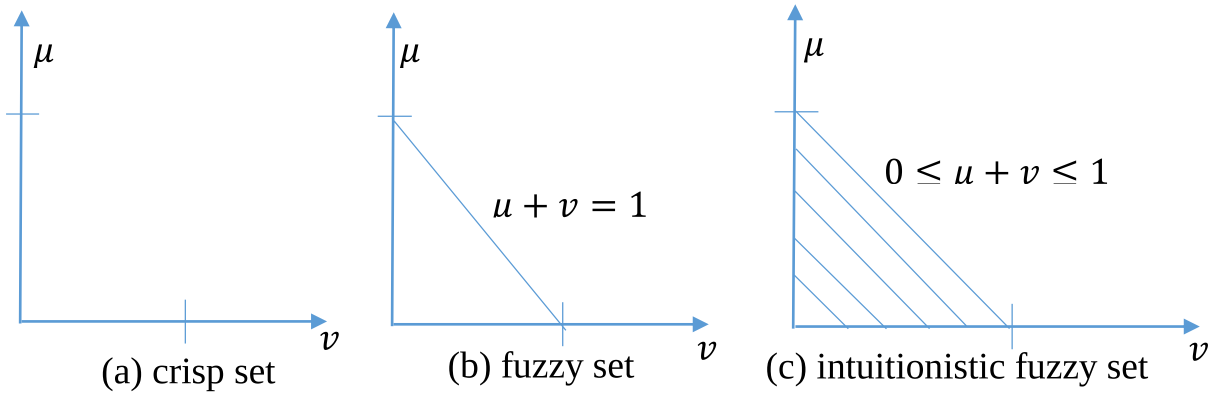


Fig. 9: Crisp, fuzzy and intuitionistic fuzzy sets on a coordinate system

In general, intuitionistic fuzzy sets (IFS) differ from fuzzy sets in terms of the approach in that IFS introduces three functions that express the degree of membership, non-membership and hesitancy [95]. The IFS approach gives a different dimension to human decision modelling by introducing three states of fuzzy constructs to characterize the extent to which decision-makers support, oppose and are hesitant or neutral about their decisions [35]. This approach helps to quantify the extent of satisfaction, dissatisfaction and hesitancy in a decision maker's judgement. Therefore, intuitionistic fuzzy sets according to [6] offers an additional degrees of freedom for decision makers through the introduction of non-membership and the hesitancy degrees.

5.4 Fuzzy and Intuitionistic Fuzzy MCDM Methods

Fuzzy sets have proven a potent method in extending multiple criteria decision making (MCDM) into decision problems where generally deterministic MCDM methods are deemed inappropriate. Fuzzy MCDM basically use linguistic values to assess or rate alternatives against sets of criteria. In most cases, the importance weights of the criteria is also determined linguistically. Fuzzy multi-criteria decision making methods (F-MCDM) become more relevant when a decision maker is faced with a challenge of quantifying linguistic statements in an otherwise deterministic decision making [2]. An example is attempting to compute the jth alternative in a project manager selection problem where 'adaptability' of a candidate is one of the criteria considered. Measuring a subjective criteria such as 'adaptability' in this real-world example poses a number of challenges including how to rate and efficiently model the linguistic ratings into a composite decision. The extension of deterministic MCDM methods with fuzzy set concept has been widely used in many disciplines including areas that were never thought of as applicable. The fuzzy extension has been utilized in almost all the methods under the domain of full aggregation and outranking methods. Some of these MCDM methods whose extension into fuzzy sets have received worldwide acclaim in terms of the number of applications are AHP,

TOPSIS, VIKOR, ELECTRE, PROMETHHE, ANP, DEMATEL, SAW among many others.

Similar to the use of fuzzy set theory in MCDM, Attanasov's introduction of intuitionistic fuzzy set has also been extended to cover some of the major MCDM techniques mentioned above. Additionally, contributions that employ either fuzzy set theory or intuitionistic fuzzy set are also known to often combine a number of MCDM methods into what is simply known as hybrid methods. These hybrid methods have over a period become popular in F-MCDM and IF-MCDM applications in diverse disciplines such as in Science, Engineering, Business, Management, Accounting, Energy, Health, Environmental Science, Arts, and Humanities among others. A hybrid method in fuzzy MCDM typically involves the use of an MCDM method to set the criteria weight and a different method for the ranking of alternatives. The next section delves into the use of hybrid methods in fuzzy multi-criteria decision making approaches. Hybrid F-MCDM methods are investigated against single method solutions to understand the rationale behind the use of such integrated approaches. The investigative piece would be followed by recommendations of appropriate use of hybrid methods. For example, the author demonstrates which MCDM methods are appropriate to be hybridized and under what peculiar circumstances. Finally, a novel hybrid F-MCDM method of Conjoint Analysis and Intuitionistic Fuzzy TOPSIS are introduced to demonstrate its applicability in case studies. The proposed hybrid method is particularly useful in situations of merging decisions of large decision group with relatively small decision body.

- **INVESTIGATIVE PART**

6 HYBRID MCDM METHOD SOLUTIONS

The concept of hybrid as relating to MCDM methods has been approached differently by different researchers. However, no matter the distinction, one basic idea holds; dealing with a combination of two notions in a decision problem. Typically the term hybrid is used in two ways to (1) describe the nature of the problem and (2) the nature of the solution (methods).

In terms of the nature of the underlying problem, some definitions given to the concept of hybrid in MCDM would include problems of incommensurable unit of measurement, combination of qualitative and quantitative criteria as well as combination of deterministic and probabilistic criteria in one decision problem [72].With regards to incommensurable units, an MCDM method is considered hybrid when different criterion assumes different unit of measurement in the same decision problem. For instance in a car selection problem, whiles the unit of measurement for fuel economy may be miles per gallon (mpg), the price would be measured in dollars (currency). The concept of hybrid in MCDM is also sometimes used to describe situations where there is a blend of both qualitative and quantitative criteria in one decision problem. Thus, a criteria with a deterministic value is combined with ones that are described with subjective ratings. An example is assessing the performance of cars using price (quantitative) and comfortability (subjective) in the same decision problem. Another idea of hybrid is when deterministic and probabilistic criteria are found in a same decision problem. In car selection problem, the price of the car is deterministic, however, fuel economy is considered to assume a random value. This is because, values for fuel economy would change based on weather, traffic and general road conditions (e.g. ruggedness of the road).

On the other hand, when the term hybrid is used to describe the kinds of methods used to offer solution to a decision problem in MCDM setting, it often refers to the use of more than one different MCDM methods used at various stages in the decision making processes. For example, the ANP method may be used to determine the weight of the criteria whiles TOPSIS is used to rank the alternatives. In research articles classified as hybrid, authors typically use words such as *hybrid*, *two-stage*, *combined* or *integrated* to describe situations where more than one MCDM method or a combination of an MCDM method and other techniques were used to solve a problem. This approach of combining different MCDM methods is premised on the series of stages in a typical structured decision-making process. Typically, the processes and steps in structured decision making include identification of the problem, construction of preferences, weighting of criteria and decision makers and ultimately, evaluation of alternatives to determine the best alternative [1]. In view of this, most hybrid, two-stage, combined or integrated methods tend to propose the use of different MCDM methods for accomplishing different tasks in the decision making process. This would often include but not limited to, the use of different MCDM methods in such stages as, setting criteria weights, aggregating decision makers' preferences, analyzing consensus in group decisions and ranking the alternatives. In this dissertation, mere extension of a deterministic MCDM method is not recognized as a hybrid solution.

In literature, some authors combine as many as four different MCDM methods to solve a problem. For instance in [96, 97] four different MCDM methods were used at different stages in the decision process. However, on the average two-method hybrid solutions tend to be the most common. As the popularity of combining different MCDM methods soar, some of the highly cited hybrid fuzzy MCDM articles are [98] where DEMATEL, fuzzy ANP and fuzzy TOPSIS methods were combined to evaluate green suppliers in a selection problem. In [99] factor analysis method and DEMATEL were combined to evaluate intertwined effects in e-learning programs whiles in [100] DEMATEL based on ANP (DANP) was combined with VIKOR in a vendor selection problem in a material recycling sector. It must be noted that, increasingly, some of the hybrid methods use other methods outside the domain of MCDM and combined them with an MCDM method. Some of these instances are seen in [99] where factor analysis, a statistical technique, was combined with DEMATEL, an MCDM method. In [101] the balance score card method which is a strategy management tool was combined with AHP, an MCDM method. Other examples of such hybrid approaches are seen in the use of quality functional deployment (QFD), SWOT analysis [103] and Delphi method [104] among many others. This dissertation uses this approach by demonstrating the use conjoint analysis method, which is outside the domain of MCDM and combining with intuitionistic fuzzy TOPSIS method in special cases of merging the decisions of large groups into small decision making groups.

6.1 Hybrid MCDM versus Single-Method Solutions

The potential for combining several MCDM methods to accomplish a task has been an interest to many researchers especially in the past decade. Figures 11-12 show current trends in the use of hybrid MCDM methods against single method solutions. It is realized that in all instances, there appears to be a gradual rise in the number of articles that use hybrid MCDM methods over single method solutions. In this instance, single-method solutions refer to when only one MCDM method was used to solve the entire decision problem. An example is using only TOPSIS or AHP in a supplier selection problem.

The first part of this dissertation investigated the usefulness of hybrid methods. In particular, decision problems in selected fuzzy hybrid MCDM articles were investigated against those that use only single methods. This was intended to ascertain whether the ranking order would be the same in both cases.

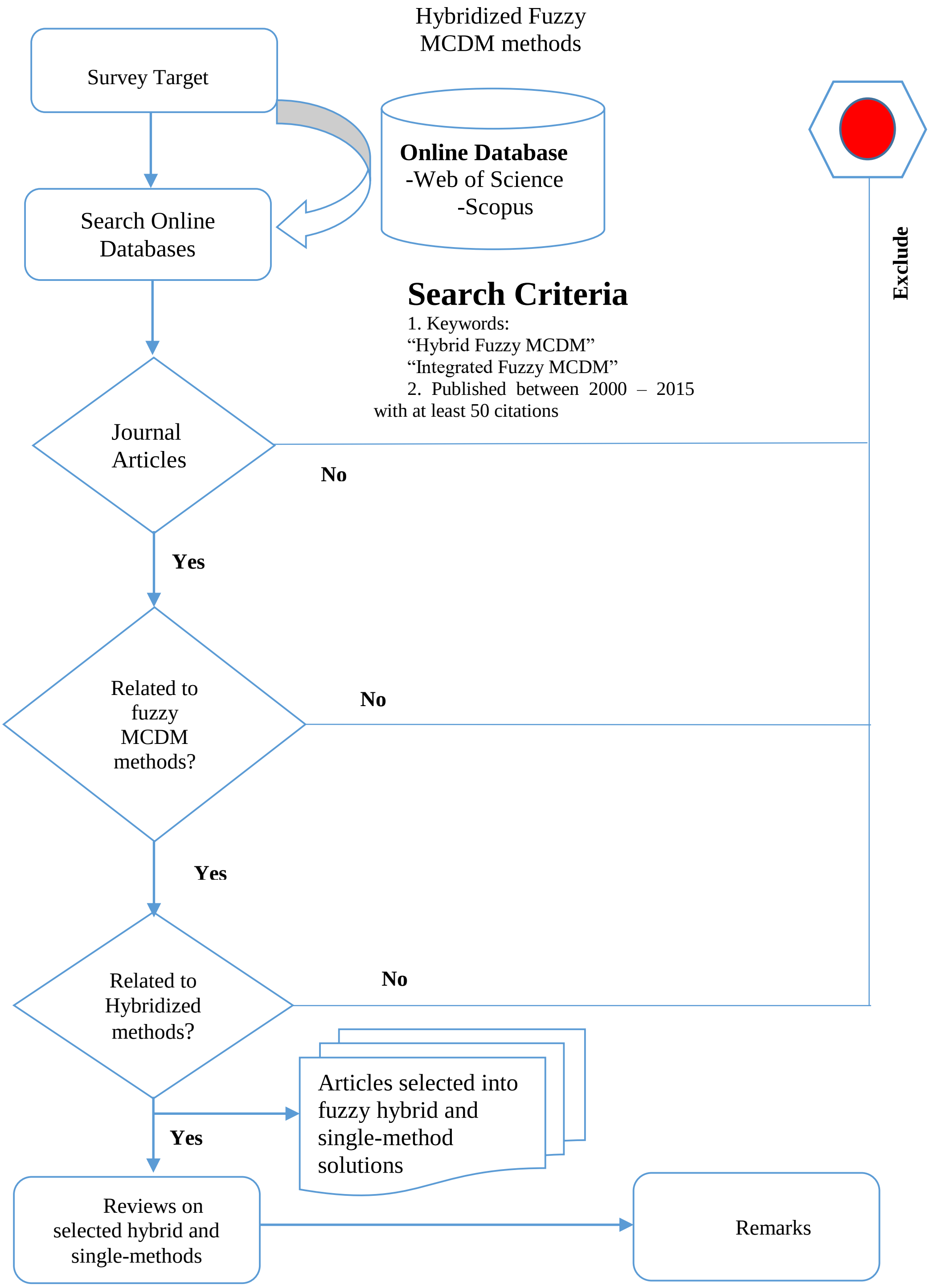


Fig. 10: Methodological concept for hybrid fuzzy MCDM literature search

To do this, the research design as shown conceptually in figure 10, sought to primarily compare the ‘decision’ output of fuzzy MCDM hybrid methods against MCDM of single-method solutions. A basic classification scheme was first created to group publications into fuzzy hybrid methods and single-method solutions. Four methods under full aggregation and 3 under outranking methods were selected for the comparison. The key requirements for the selection of each of the MCDM methods under *full aggregation* and *outranking* methods was by their (i) automated search availability (ii) popularity among researchers and (iii) indexation in Scopus or Web of Science database. In figures 11 and 12, which are grouped into *full aggregation* and *outranking methods*, there was a strong evidence of a yearly rise in the use of hybrid MCDM methods as opposed to single-methods. In all instances in figures 11 and 12, hybrid methods which were virtually non-existent in the early years of the last decade, were seen to be upstaging single fuzzy MCDM methods in later years of the decade.

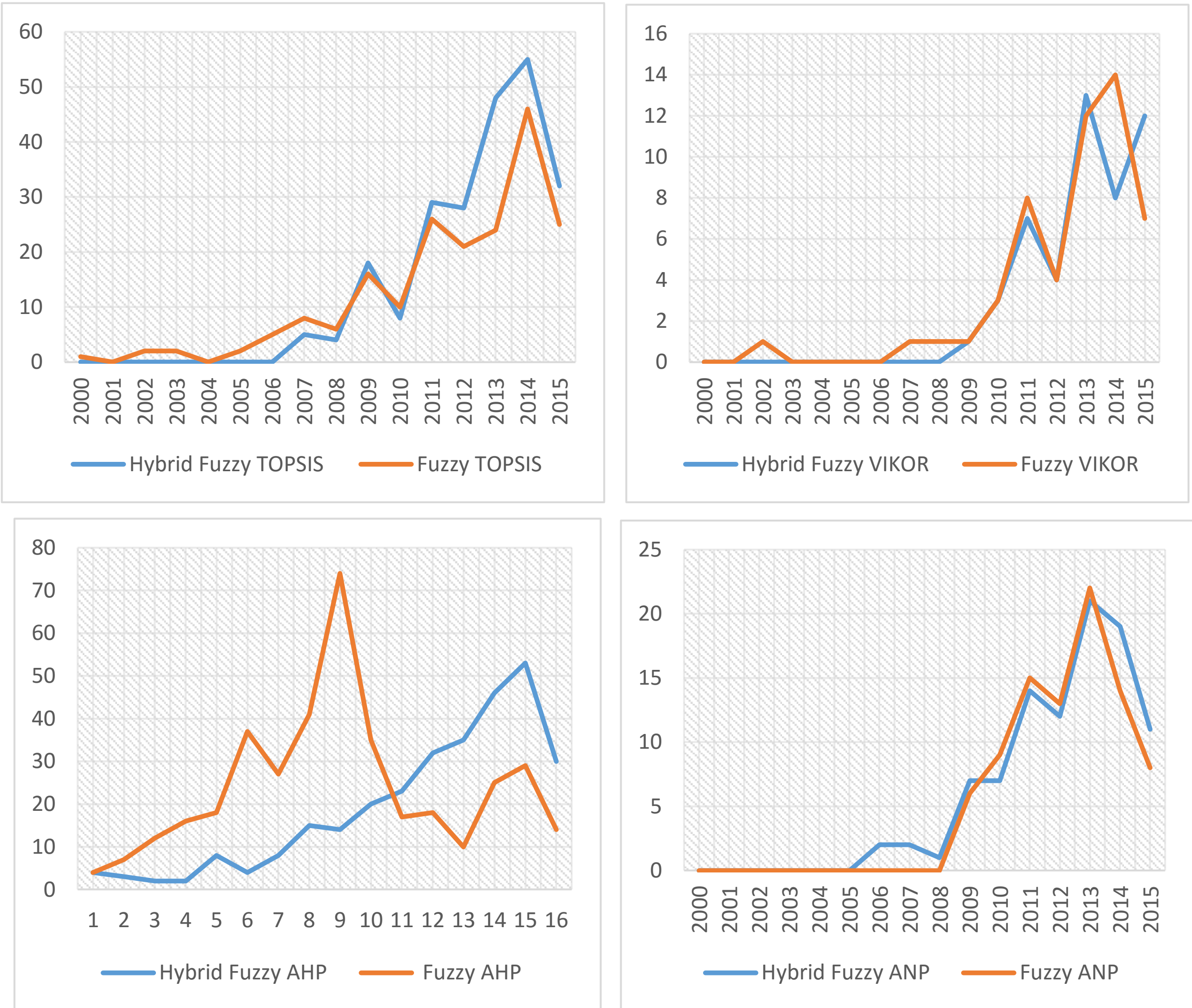


Fig. 11: Full Aggregation Methods: Trend of research between hybrid MCDM and single methods, 2000-2015

Following the revelation that fuzzy hybrid MCDM methods are becoming more popular among researchers than the use of single MCDM methods, a further investigation was carried out to ascertain whether hybrid methods were used appropriately. In particular, the research sought to find out whether in decision problems for which hybrid methods are employed, single-method solutions could also be used to realize the same ranking order. To carry out this task, 30 research articles that meet the requirements in figure 10, were selected for the task of comparing hybrid MCDM methods with single-method solutions on a same decision problem. Finally 10 of the articles were selected based on the fact that, they provided numerical examples of DMs ratings to be able to apply both hybrid and single MCDM methods on the same problem. As shown in the second and fifth column in table 4, the original hybrid method used in the research paper are compared with the decision solution using a single-method in the hybrid approach.

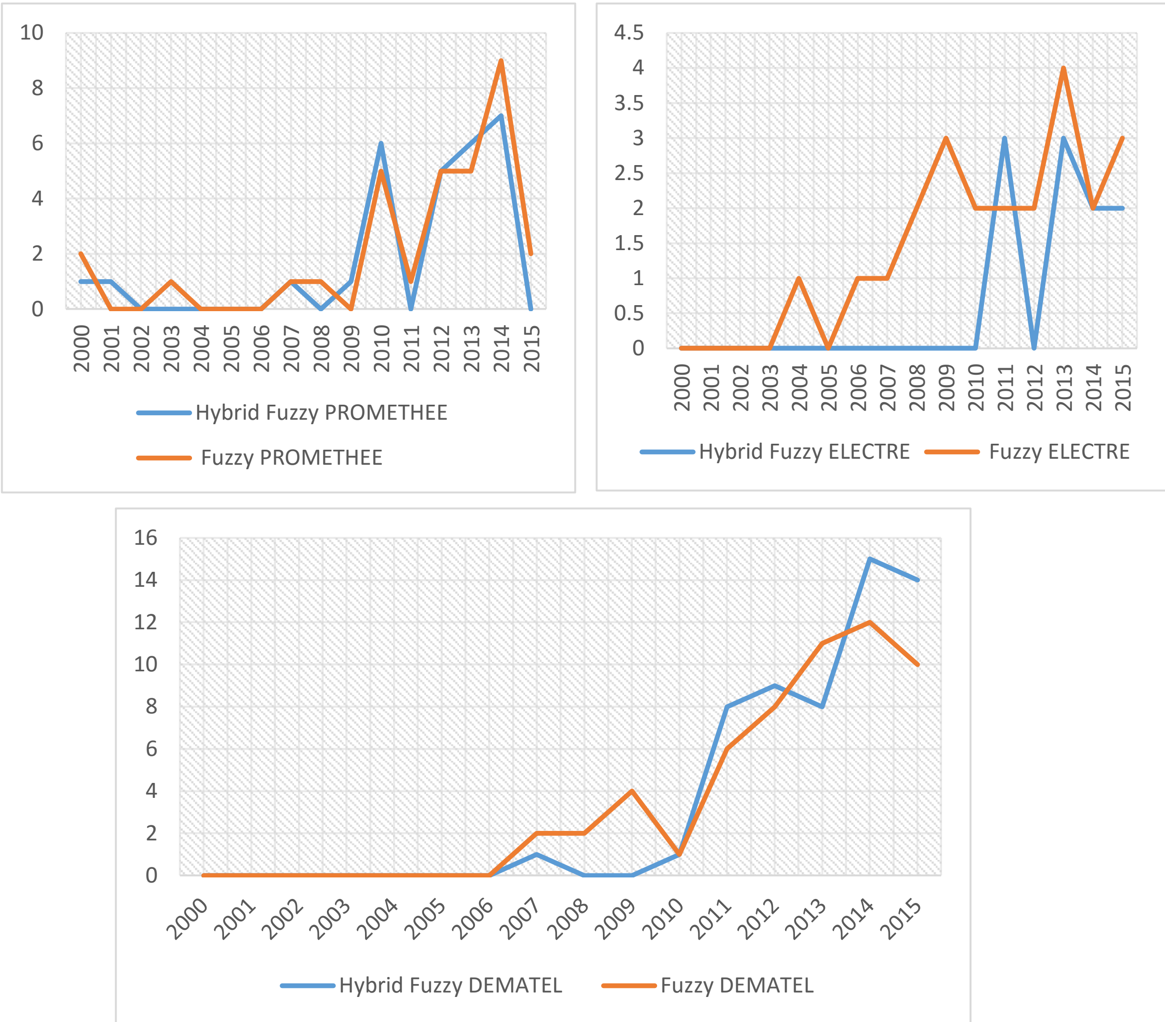


Fig. 12: Outranking Methods: Trend of research between hybrid MCDM and single methods, 2000-2015

The results on the new ranking order as a result of using a single MCDM method to solve the same decision problem, are then flagged as ‘change’, ‘no change’ or ‘irresolution’. A ‘change’ means when the single-method was used, the same ranking was not achieved as in the original. On the other hand, a ‘no change’ means the same ranking order is achieved whether a hybrid or just one of the methods in the hybrid is used. An irresolution is recorded when the nature of the decision problem requires some special treatment of the criteria weights which may produce different ranking order with different single MCDM methods. The results from the pilot study though not exhaustive of all hybrid fuzzy MCDM solutions in literature, showed that out of the 10 sampled articles under the investigation, 6 of them produced the same ranking order as shown in Table 4 below.

Table 4. Results of ranking order between hybrid and selected single MCDM methods.

Author	Methods used in the Hybrid	Number of criteria used	Number of Alternatives	Single MCDM method used	Ranking order
[105]	Fuzzy DEMATEL + Fuzzy TOPSIS	17	4	Fuzzy TOPSIS	Change
[106]	Fuzzy AHP + Fuzzy TOPSIS	23	5	Fuzzy TOPSIS	No change
[107]	Fuzzy AHP + Fuzzy TOPSIS	28	21	Fuzzy TOPSIS	No change
[108**]**	Fuzzy AHP + PROMETHEE	7	4	Fuzzy PROMETHEE	No change
[100]	Fuzzy DANP + Fuzzy VIKOR	17	6	Fuzzy VIKOR	Change
[109]	Fuzzy ANP + Fuzzy ELECTRE + Fuzzy TOPSIS	10	6	Fuzzy ELECTRE	Irresolution
[110]	Fuzzy ANP+Fuzzy DEMATEL+ Fuzzy ELECTRE	69	4	Fuzzy DEMATEL	Irresolution
[111]	Fuzzy AHP+Fuzzy SAW	14	6	Fuzzy AHP	No change
[112]	Fuzzy SWARA + Fuzzy VIKOR	7	4	Fuzzy VIKOR	No change
[113]	Fuzzy ANP+Fuzzy TOPSIS	31	5	Fuzzy TOPSIS	No change

For instance in article [107], the original hybrid method used fuzzy AHP and fuzzy TOPSIS in the hybrid. However, when only fuzzy TOPSIS was applied as a single method, there was no change in the ranking order. The results in Table 4, raises issues about the appropriateness of hybrid MCDM methods as raised by other researchers [54, 55, 56]. It is observed that whiles hybrid MCDM methods are quite useful especially for very complex decision problems, determining the appropriate use for an ideal problem poses a challenge.

The investigation further revealed that when two or more full aggregation methods (American school) are used in a hybrid approach and subsequently, a single MCDM method in the hybrid is used on the same decision problem, the same ranking order is most often achieved. For example in the sample articles [106, 107], both AHP and TOPSIS belong to full aggregation methods. When these two methods are combined in a hybrid method and then subsequently one of the method is used on the same decision problem, the results tends to be the same particularly when there are no designated ‘cost’ and ‘benefits’ criteria. Where there exists a cost and benefit criteria with dependencies, a use of TOPSIS realizes the same ranking order. Similarly when the two methods used in the hybrid belong to the category of outranking methods (French school), then a subsequent use of only one of the methods in the hybrid solution tends to achieve the same results.

It was further observed that the ranking order with a single MCDM method as compared with a hybrid method, tends to be different when the two methods come from different categories or types of MCDM methods. For instance when one of the methods in the hybrid comes from a full aggregation background and the other comes from an outranking method, the results in terms of the new ranking tends to change. Thus meaning the ranking order becomes different as seen in sampled articles [100, 105].

Investigation into the use of hybrid MCDM methods, inspired the proposition in this dissertation of a novel hybrid fuzzy MCDM method ideally suited for special situations of merging separate decisions from a large group of DMs on one-side, and a relatively small group from another. In the following sections, the proposed hybrid fuzzy MCDM method is introduced. In the approach, conjoint analysis is used to model preferences of a large group of decision makers (DMs) and converted into criteria weights. The criteria weights decided by the large group are now factored into another decision making level involving a small group of decision makers. Intuitionistic fuzzy TOPSIS method is the other method in the hybrid, where it is used to intuitively model expert ratings into a ranking order in a selection problem.

In the following section, the concepts of weights setting in MCDM is briefly explained. An understanding of the role of weights in decision making as well as how they are set, sheds light on the proposition and use of conjoint analysis to set criteria weights as fully explained in chapter 8.

- **THE CONCEPT OF ‘WEIGHTS’ IN MCDM**

7 WEIGHTS USAGE IN MCDM

The notion of assigning weights, either to criteria or decision makers is central to most multi-criteria decision making (MCDM) methods, and is used mostly during preferences aggregation stage. The most popular form is when criteria are weighted essentially to determine their relative importance to the overall decision making goal. In this dissertation, one of the goals is to find an appropriate method of assigning weights to a set of criteria as demonstrated in chapter 8. It must be noted that whereas some MCDM methods, depending on the decision problem assume an equal weights or importance for all the criteria, most usage of criteria weights readily assumes that some criteria would contribute more to the composite decision than others. With a host of aggregation algorithms in MCDM, several different methods of determining criteria weights have been proposed over the years. Literature is abound with many of such weighting methods where the main purpose is to assign either a cardinal or an ordinal value to the range of criteria to distinguish them in terms of their contributory value to the final decision. MCDM practitioners consider weight setting as one of the most important steps in the decision making [152]. However, with a wide array of methods and approaches, weight assignment is sometimes abused by a misuse of weight setting methods. In view of this, the choice of an MCDM method especially for such task as weight setting, should always be guided by what the decision analyst wants as an output, and what the method can help achieve. For example, the intended meaning for using AHP (a full aggregation method) or ELECTRE (an outranking method) are completely different. In the weighted sum for example, the weights of criteria are intended as tradeoffs whiles in ELECTRE methods, weights are considered as voting power of the criteria considered. Another important observation one must consider, is when combining for example, AHP-ANP with Choquet integral preference model. The Choquet integral is generally used in special cases where there is an interaction among criteria and a classical additive value function cannot be applied to aggregate the performances of the alternatives on the considered criteria. To this end, a thorough classification of weight setting methods helps to choose an ideal method for the right problem.

In ranking MCDM methods, to which this dissertation focuses, weighting methods can simply be grouped into 3 kinds; subjective weighting, objective and hybrid weighting methods. The hybrid is used in rare but useful situations where the problem demands the use of both subjective and objective weighting methods. In subjective methods, criteria weights are set based on decision makers' ratings or preferences. For example in Table 5 below, subjective ratings of decision makers help decide the importance of each of the criterion through the aggregate sum of their preferences. In this instance, criteria 3 according to Table 5 is therefore deemed relatively the most important. The ratings here are *Very High (VH), High (H), Medium (M), Low (L) and Very Low (VL).*

Table 5. Sample of subjective criteria importance weight

	D_1	D_2	D_3	D_4	D_5	Aggregated Weight	Priority/Ranking
C_1	H	H	H	VH	H	$\langle 0.796,0.168\rangle$	2
C_2	H	M	H	M	M	$\langle 0.667,0.280\rangle$	3
C_3	VH	VH	VH	H	VH	$\langle 0.939,0.059\rangle$	1
C_4	VL	M	L	M	L	$\langle 0.318,0.629\rangle$	6
C_5	L	L	H	M	M	$\langle 0.456,0.486\rangle$	5
C_6	M	H	M	H	M	$\langle 0.638,0.310\rangle$	4

Some of the key subjective weighting methods widely used in literature are the pairwise comparison (AHP, ANP) methods, trade-off methods and the Delphi method among others. Since subjective criteria weights are set by decision makers, the weights can be influenced to reflect an acquisition of a new knowledge or changes to requirements of the decision goal.

Objective weights on the other hand are derived using existing or historical data with the help of mathematical models [153]. Objective weighting methods include multi-objective optimization methods, TOPSIS, the Shannon entropy method, principal component analysis (PCA) among others. They are deemed relevant in MCDM especially when reliable subjective weights are difficult to obtain or non-existent. Computation of objective weights are sometimes beset with problems of accuracy and reliability [153].

The third approach to weights setting in MCDM is the use of hybrid methods that include the combination of additive and multiplication synthesis [153]. In figure 13, is a summary of some methods and approaches that are used in subjective, objective and hybrid weights assignment in MCDM. It must be noted that the conjoint analysis method used to set weight in this work, is a trade-off method which comes under subjective weighting methods.

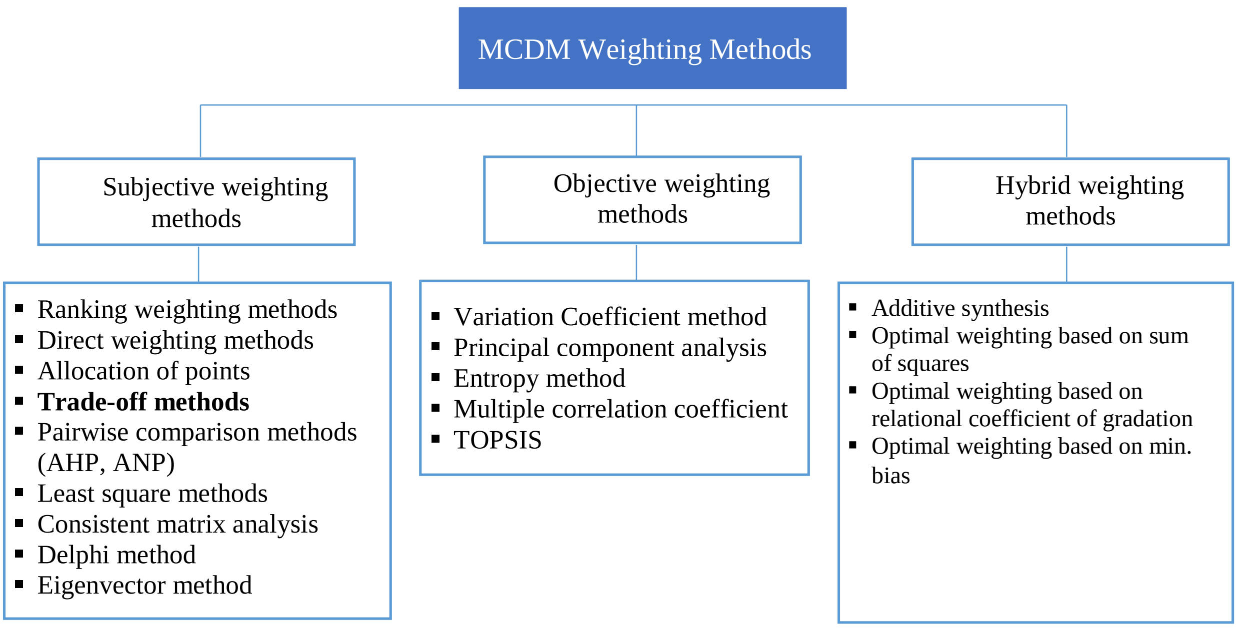


Fig. 13: Hierarchical outline of MCDM methods (source: [153])

7.1 Choice of a Weighting Method

The decision to select an appropriate weighting method for multi-criteria decision problems, is one of the most difficult processes in the chain of steps in MCDM. This is because for MCDM problems where weights assignment is required, the weights become the fulcrum to the final ranking and selection of the competing alternatives. Thus a slight change or inaccuracies in the weights can have a congruent effect on the final ranking results. With this in mind, and with Hobbs's [154] research affirming that different weighting methods result in different set of criteria weights and consequently on the overall results, care must be taken to choosing an appropriate weighting method suited for a particular decision problem. In furtherance to the results in [154], [155] deployed five weighting methods to assign ranges of weights to sets of criteria. The findings as a result of comparisons among the various weighting methods suggested that in subjective weighting methods, decision makers would normally assign almost the same weights scores to criteria even with different weighting methods. However, in terms of the suitability of the weighting method and its ease of use to the decision problem, some minor variations were recorded [155] as cited in [153]. These slight inconsistencies in weights values derived with deferent weighting methods have the potential of altering final results especially when alternative performance scores are so close to each other.

In view of this, it is obvious that no single weighting method in MCDM is adequately enough for all decision problems. However, the strength of each weighting method must be brought to bear on the decision problem at hand. For instance [156] found no weighting technique overly superior to the others and therefore asserted that a preferred weighting method in translating experts' ratings into relative weights must always be based on the intended use of the scale (ordinal, interval, or ratio level of measurement) to be used. Other considerations for choice must also look at the specific decision problem at hand, ease of use of the method especially computational time involved, respondents (decision makers) adequate knowledge and understanding of the requirements of the weighting method and the general output expected from the decision problem.

In [157] MCDM weighting methods are classified into two broad categories; direct and indirect estimation. In direct estimation, questionnaires are mostly used to elicit preferences expressed in relative importance of the attributes where respondents' priority statements in terms of their responses are expressed in numerical terms. The following are some of direct weighting estimation techniques [153].

- **Rating method**: Rating related methods basically require respondents (decision-maker) to determine the relative importance of each criterion in a set of criteria (attributes) by distributing numerical values of points among the criteria.

- **Ranking method**: In ranking methods in direct estimation, respondents on the other hand rank a set of criteria in order of their importance.
- **Trade-off method**: In trade-off weighting methods, the decision-maker by assigning a relative importance value to a criterion trades off one quality or aspect for the other. This is done by directly placing weights on a set of criteria in a pairwise combination respect to all other criteria [153]. Weighting methods that rely on trade-offs are widely used because of the strong theoretical foundation they possess [153]. Since trade-off method makes compromises between two criteria, a decision maker does not necessary assign weights or determine the relative importance of a criterion directly [154]. The decision maker is only allowed to state the extent of compromise they are willing to make between two criteria when the two criteria cannot be chosen at the same time. However, by making their preferences through trade-offs, decision makers' preferences indirectly aggregate into relative importance of the criteria. Conjoint analysis technique is one such trade-off method though not traditionally an MCDM method. Among MCDM methods, AHP is noted as the most reliable trade-off method.
- **Pairwise comparison**: In paired comparison, the goal is to basically address issues of unfairness by satisfying the Condorcet criterion. This is carried out by pitching the performance of each alternative in head-to-head with each of the rest of alternatives. Pairwise comparison is widely used because of its strengths over other direct weighting estimation methods which become inefficient when number of objectives grow large [158] as cited in [153]. The problem with large number of objectives are adequately dealt with the AHP method, which both concurrently provides pairwise comparison and trade-off function. Pairwise comparison is found to provide clear interpretable results which are easy to produce unlike trade-off methods. The method works better with finite alternatives. The methodology is found to be the most widely used for assigning weights when the relative importance of criteria is of utmost importance. However, pairwise comparisons show some weaknesses in its preference modelling. The most obvious one is its inability to allow respondents to express a separation distance in their choices of best and worst alternatives. A second concern about pairwise comparison methods is the seeming complexity that characterizes the comparison of objects in pairs especially when there are large sets of criteria and respondents involved [154]. Lastly, there is the inconsistency at the level of the decision maker concerning pairwise comparison especially when there are large number of criteria involved [153].

According to [157], in indirect weighting estimation methods, weights are assigned based on respondents' choices in the past. In essence, weights are derived through the estimation of previous behavior or preferences of decision-makers. It is observed that both direct and indirect weighting methods possess strengths and weaknesses in terms of accuracy that make them applicable only in unique situations. In other words, the usefulness of each method depends largely on the input time, the attitude and conditions of the decision-makers [158], [159]. In [154] where two weighting methods were used in plant location problems, the result produced completely different values. In the experiment, trade-offs and a rating method were explored which produced different weighting values. This affirms other research by [155] that, different weighting methods produce different results, hence the need to select an appropriate weighting method for particular problems.

In practice, the two most widely used weighting methods are those that fall under pairwise comparison and trade-off methods. Weighting methods that allow for trade-offs to be made by decision makers' preferences consistent with the underlying decision rule, tend to yield reliable and consistent weights [153]. Trade-off weighting methods though widely accepted, tends to be time consuming and difficult to implement in practice. The next most widely used weighting MCDM method are those that belong to the range of pairwise comparison methods. AHP is one method that provides both functionality of trade-offs and pairwise comparison and therefore a preferred choice in weight setting in MCDM.

In this dissertation, conjoint analysis, a method outside the scope of MCDM but which functions almost the same as most MCDM methods is used to set weights. It is demonstrated that although AHP is a preferred choice for trade-off modelling in MCDM approaches, conjoint analysis has some inherent advantages over AHP especially in terms of the number of input preferences it can model. Additionally, conjoint analysis with its partial and total part-worth utilities also easily reveal preferences at every level of the attributes, a strength that AHP lacks. Furthermore, conjoint analysis is also best suited for decision problems investigated in the numerical sections of this work. In particular is its ability to adequately model preferences of a large group of people and transform the relative importance of each attributes (criteria) into criteria weights. In the following sections, the proposed hybridized decision framework is introduced. Conjoint analysis for criteria weight setting and intuitionistic fuzzy TOPSIS method for alternatives ranking are presented.

- **PROPOSED HYBRID/INTEGRATED METHOD**

(Conjoint Analysis - Intuitionistic Fuzzy TOPSIS method)

8 A TWO-TIER HYBRID MCDM MODEL

The investigative section provided insights into the appropriate use of MCDM methods especially when used in special cases in a hybrid structure. This section proposes a novel hybrid decision making method appropriate in situations of merging decisions from two different groups of decision makers. The basic notion is to model decision situations where the preferences of a large group of decision makers are expressed as criteria weights and subsequently factored into another level of decision making involving a small group of DMs selecting a best alternative from a competing set of alternatives. The two levels of decision making are conveniently described as a two tier hybrid MCDM model. The two-tier model is especially ideal in many cases where preferences and opinions of a large group of people (for example, shareholders, customers, employees, students etc.) are merged into a final decision making stage involving small group (experts) (e.g. board of management, marketing executives, managing directors, school management board etc.). Such two-tier decision making structures arise when a higher decision making body made up of a relatively small group of experts, solicits opinions from a lower decision making body made up of a relatively large group of people (more than 100). For example, in a hypothetical situation where a company's executives want to select an ideal location to site a new sales branch, customers (large group) preferences and trade-offs may be factored into such decision making structure. In this regard, the preferences of customers can be modelled and incorporated into the final decision by the executives.

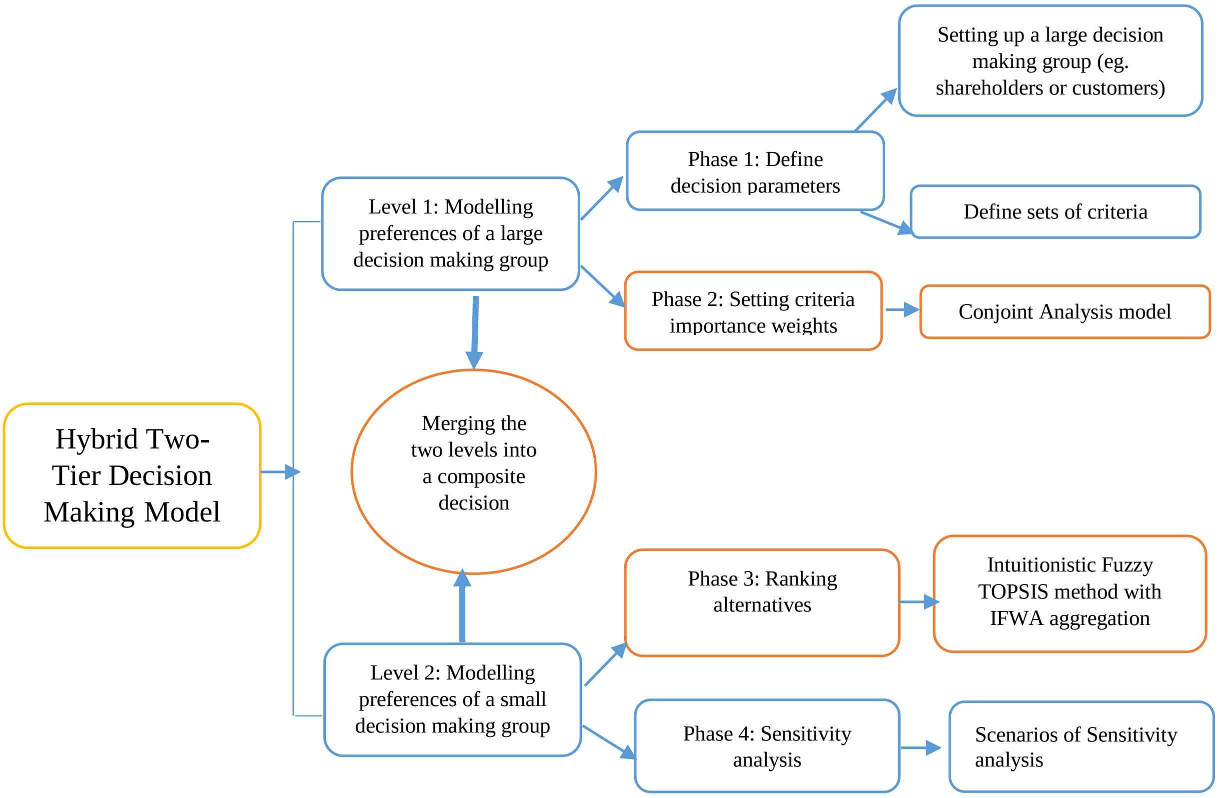


Fig. 14: Schematic diagram for a two-tier hybrid MCDM decision making model

In this dissertation, two existing methods (one MCDM method and a popular marketing research tool) are employed to demonstrate the modelling of such two-tier decision problems of incorporating two decision points into a composite one. Whiles many MCDM methods can be used to set criteria weights (as described in chapter 7), the special case of modelling preferences and trade-offs of large group of decision makers, can ideally be performed using conjoint analysis. Conjoint analysis is used to elicit preferences of the large decision making group and treated as criteria weights. Intuitionistic fuzzy TOPSIS method on the other hand, is used to rank competing alternatives. The conjoint analysis method which is traditionally not considered an MCDM method shares similar features with full aggregation MCDM methods. In the following section, the conjoint analysis method is compared to MCDM methods where its use in two-tier model is justified.

8.1 Conjoint Analysis Technique and MCDM Methods

Conjoint Analysis (CA) and Multi-Criteria Decision Making Methods (MCDM) in general, share similar characteristics especially in the way they function, even though they both have different objectives. For instance, presented with a problem, both methods typically decompose the problem into a set of criteria or attributes with several levels. Additionally, they both typically employ linear additive functions to determine the aggregate relative preferences of a given set of alternatives [114]. However, these similar features in conjoint analysis and MCDM have rarely been explored together by researchers and practitioners. One of the several reasons identified for this lack of collaboration could be their traditionally restricting areas of application. Whiles conjoint analysis is used to elicit customer preferences to predict their purchasing decisions in marketing research, MCDM is popular in decision sciences and operations research for evaluation, ranking and selection problems. Perhaps the clearest challenge with deploying conjoint analysis and MCDM techniques is the size of data conventionally used in both methods. IJzerman et al., [114] explains that typically, CA approaches require large data sets whereas MCDM methods, for example Analytical Hierarchy Process (AHP), are capable of working with a single decision maker. In view of the seeming challenges in utilizing CA and MCDM methods together, this paper demonstrates an applicability of the two methods as a 2-tier hybrid decision method to incorporate preferences of large decision making group into a relatively small decision making group.

The paper recognizes some contributions of decision making models that have created the research space for the concurrent use of conjoint analysis and multi-criteria decision making methods. A non-technical article by [115] gave an overview of utilizing conjoint analysis tools in multi-criteria decision making methods. Scholl, et al. [116] compared solving multi-attribute design problems with analytic hierarchy process and conjoint analysis and found both useful for modelling respondents' preferences. Peniwati [117], evaluated several group decision making methods and found conjoint analysis as a potent technique in terms of its depth of analysis and 'faithfulness of judgments'. Subsequently [114] compared the performances of AHP and conjoint analysis in the assessment of treatment alternatives for stroke rehabilitation. IJzerman et al. [114] identified several similarities and differences between the two methods. Literature is abound with research contributions where various forms of user preferences were explored in decision making problems using conjoint analysis and MCDM methods. However, combining conjoint analysis and an MCDM method has rarely been explored in literature. This dissertation demonstrates the appropriate use of conjoint analysis and an MCDM method in special cases of merging two independent decisions from two different decision levels. A demonstration of such applicability of conjoint analysis-MCDM is shown with 3 numerical examples in the case study section. In view of the prospects, challenges and general progress made in modelling user preferences using various methods, this dissertation proposes a 2-tier hybrid conjoint analysis – intuitionistic fuzzy TOPSIS framework to incorporate large decision group preferences and trade-offs (which culminate into criteria weights) into a relatively small decision group (experts). The conjoint analysis is comparatively ideal for modelling large decision making groups. Intuitionistic fuzzy TOPSIS is ideally chosen for ranking alternatives because of its ability to decipher how much of a decision maker's rating is satisfactory, dissatisfactory or hesitant. This helps to put premium or not on a decision maker's judgements. The following sections explain the proposed hybrid methodology, divided into first and second tiers and a thorough background to the two methods explained.

8.2 First Tier - Conjoint Analysis

Conjoint analysis (CA), widely utilized in marketing research, originated from mathematical psychology with introductory articles by [118,119,120] among others. However, before these papers, [121], [122], and [123] as cited in [124] had made significant contributions toward observing decision makers' (DMs) policies and decisions over time and making inferences from them. Later on, contributions from [125] and more recent contributions by [126], [127] and [128] have helped to improve trade-off decision analysis using conjoint analysis. Conjoint analysis technique basically models preference data of judges who are predominantly non-experts (service users, shareholders, consumers, potential customers, etc.) on a range of stimuli (products or services) by their attributes, where the attributes have different levels [126]. The technique works by decomposing the preference data into a so-called part-worth utilities and examining the trade-offs of the decision makers (DMs) between different levels. This approach helps to determine the relative importance DMs attach to attributes and their levels [116]. Over the years, the robustness, accuracy and reliability of CA has been vouched in literature through studies by [129], [130], and [131] as cited in [132]. Conjoint analysis is typically a simple-effects Analysis of variance (ANOVA) that produces some specialized outputs. The model in Eq. (39), expresses a decision maker's stated preferences for a range of attributes at various levels in a full factorial design.

$$y_{x_1 x_2 \cdots x_n} = \mu + \sum_{i=1}^{n} \beta_{x_i}^{A_i} + \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \beta_{x_i x_j}^{A_i A_j} + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k \neq i}}^{n} \beta_{x_i x_j x_k}^{A_i A_j A_k} + \ldots + \beta_{x_1 x_2 \ldots x_n}^{A_1 A_2 \ldots A_n} + \varepsilon_{x_1 x_2 \ldots x_n}, \quad (39)$$

where μ is the grand or the general mean; $\beta_{x_i}^{A_i}$ is the effect of attribute A_i at the level x_i; $\beta_{x_i x_j}^{A_i A_j}$ is the effect of attributes A_i and A_j at levels x_i and x_j respectively; and $\varepsilon_{x_1, x_{1,\ldots} x_n}$ is the error term. The attributes represent the independent variables, the preferences are the dependent variable and the $\beta^{\prime s}$ are the part-worth utilities which signify the parameter estimates from the ANOVA model [133]. In this paper, the additive model is used to compute the part-worth utilities of each respondent which consequently results in the total utility attached to an attribute (criteria) by the group of respondents (large decision making group). In Eq. (40), an individual decision maker's utility U regarding an attribute (criteria) C_i is expressed in the additive format.

$$U(C_i) = \sum_{i=1}^{m} \sum_{j=1}^{m_j} \beta_{ij} G_{ij} \quad (40)$$

Where β_{ij} defines the part-worth utility estimate on the j-th level of the i-th factor (i=1, 2,…n and j=1, 2,…m), and G_{ij} indicates the presence of the j-th level of the i-th attribute or criteria (where G_{ij} = 0 if attribute j is not present or 1 if attribute j is present).

8.2.1 Relative importance of attributes/criteria

The computation of the relative importance of each attribute is a very important step in conjoint analysis and as far as this dissertation is concern. It is essentially the weight of importance assigned to the attribute (criteria) evaluated in percentages. In Eq. (41) W_j is the relative importance of attribute (criteria) C_i; Max(β_{ij}) represents the maximum utility of the attribute, meaning the most preferred attribute level whiles Min(β_{ij}) is the minimum utility signifying the least preferred level [134].

$$W_j = \frac{Max(\beta_{ij}) - Min(\beta_{ij})}{\sum_{j=1}^{t} [Max(\beta_{ij}) - Min(\beta_{ij})]} \times 100 \quad (41)$$

At this stage, the weight W_j represents the importance that decision makers (large group) assign to each of the attributes (criteria) to be used in the subsequent selection problem. The weight W_j obtained from decision makers for each of the criterion are factored into the second-tier using intuitionistic fuzzy TOPSIS method.

8.2.2 Types and Uses of Conjoint Analysis

Typically, conjoint analysis is grouped into 4 categories namely; the traditional conjoint analysis (TCA), adaptive conjoint analysis (ACA), choice-based conjoint analysis (CBCA) and self-explicated conjoint analysis [160]. The first three methods; TCA, ACA and CBCA are decompositional in the way they function, by decomposing the preference data into a so-called part-worth utilities [121], [160]. Self-explicated conjoint analysis on the other hand, functions from bottom-up in a compositional structure. Unlike the first three, it composes the preference values usually from ratings based data on attribute levels and subsequently determines the relative importance of attributes [160].

There are stark differences among the four types of conjoint analysis even though the decompositional types share similar features in the way they function. These differences serve as a guide to choosing each of them in real-life applications. The traditional conjoint analysis (TCA) which basically uses 'stated' preferences gathers and models respondents' preferences for profiles of a set of stimuli (products or services) where the entire (full) set of attributes (criteria) are utilized for the study [125],[126]. This type of preference modelling in CA is called full profile design. In practice however, using the complete set of full profiles often becomes burdensome to respondents. For example in a situation where there are even only 4 attributes each at three levels, there would be a total of (3 x 3 x 3 x 3) = 81 possible sets of profiles which practically becomes impossible to get accurate and reliable responses from respondents. In view of this, a smaller set of full profiles chosen in an experimental design, are normally used to make the work of respondents less burdensome and by extension have a highly reliable preference data [160]. Full profile design in TCA when necessary, actually mirrors real-world decision on alternatives in a realistic manner by presenting all sets of combinations in an integrated multi-criteria concept. Traditional conjoint analysis uses regression-based approaches to decompose a decision-makers' overall stated preferences into separate utility values that correspond to each attribute [160]. The separate utility values are referred to as 'attribute-specific part-worth functions' [160]. According to [160] the preference functions can in most cases be estimated at the individual level where the preference function is seen as an indirect utility function.

Over the years, the challenge faced by practitioners in utilizing the traditional de-compositional conjoint method, generated research interests in alternative workable conjoint analysis approaches. A significant number of these new data collection and design methods are based on the idea of modelling preferences generated under hypothetical scenarios that mirrors the underlying real-world problem [126]. These new approaches also proposed the use of new estimation methods for part-worth functions such as multinomial logit methods, primarily designed for choice-based conjoint analysis methods, the share model for share allocation studies and logit models for probability of purchase situations [126]. Table 7 presents a host of conjoint data collection techniques and their respective estimation approaches. Choice-based conjoint analysis methods are theoretically based on discrete choice analysis where a prediction is made on choices between two or more discrete alternatives. They are sometimes referred to simply as "stated" choice methods because they design and model discrete hypothetical choice possibilities of respondents [160]. CBCA is among one of the widely used CA methods. Primarily based on behavioral theory of random utility maximization [161] as cited in [160], the CBCA approach decomposes a decision maker's random utility for a stimulus into two parts: deterministic utility and a random component [160]. Over the years, a number of alternate models have been designed that explain the probability of choice of a stimulus (object) based on the distributional assumptions of the random component [160], [130]. The most popular and widely used of these new models is multinomial logistic regression (logit model). All these methods and approaches come under the family of discrete choice analysis methods. Table 6 presents various steps in conjoint analysis and various diverse approaches used in providing solutions.

Table 6. Steps in conjoint analysis and various solution approaches

STEP	METHODS
Choice of prefderence model	Vector model Part-worth model Ideal point model
Data Collection	Trade-off Full profile
Stimuli Construction	Fractional factorial design
Stimuli Presentation	Visual presentation Description Real product
Preference scale	Rank Score Pairwise comparison
Estimation	Multiple regression (metric) ANOVA (non-metric)

Table 7. Methods of data analysis in conjoint measurement

Forms of Preference modelling in CA	Data Analysis
Ratings	Regression Analysis
Choice	Multinomial Logit
Rankings	MONANOVA or LINMAP
Share Allocation	Share Model
Probability of Purchase	Logit Model

8.3 Second-Tier – Intuitionistic Fuzzy TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method proposed by [47], has become one of the most popular techniques in Multiple Criteria Decision Making (MCDM). Similar to all other deterministic MCDM methods, the fuzzy and the intuitionistic fuzzy extensions of TOPSIS have subsequently been developed. Basically, there are five computational steps involved in the TOPSIS approach, starting with the gathering of performance scores of alternatives with respect to the set criteria. The second step involves normalization of the performances for uniformity. The normalized scores for the competing alternatives are subsequently weighted and their distances to ideal and anti-ideal points calculated [56]. Finally, the relative closeness to an ideal solution is given by the ratio of these distances. These five steps are explained in detail with the intuitionistic fuzzy extension form in this chapter.

The above computational steps are extended using intuitionistic fuzzy sets. Like the original TOPSIS method, the intuitionistic fuzzy TOPSIS also relies on the so-called shortest distance from the Intuitionistic Fuzzy Positive Ideal Solution (IFPIS) and the farthest distance from the Intuitionistic Fuzzy Negative Ideal Solution (IFNIS) to determine the best alternative, as shown in figure 15. In figure 15, point A is shortest to the ideal solution and therefore becomes the best solution. The IFNIS maximizes the cost criteria and minimizes the benefit criteria, whiles IFPIS maximizes benefit criteria and minimizes cost criteria. The alternatives are ranked and selected according to their relative closeness determined using the two distance measures. Similarly, the extension of TOPSIS into intuitionistic fuzzy sets also maintains the key features such as the FPIS and the FNIS [20]. Intuitionistic Fuzzy TOPSIS was chosen because it is theoretically more robust [135], provides a sound logic that mirrors the human reasoning in selection of alternatives, and presents a scalar score value that accounts for both the best and worst alternatives at the same time [136].

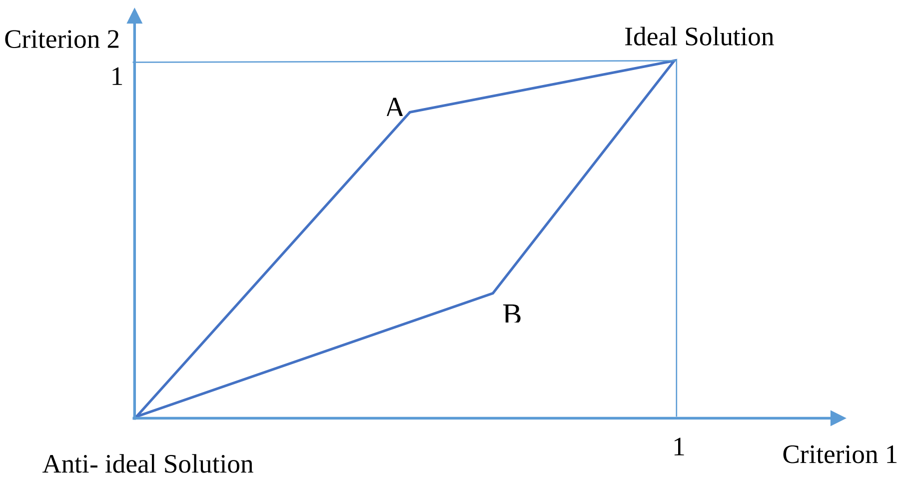


Fig. 15: Conceptual model of TOPSIS method

8.3.1 Steps for Intuitionistic fuzzy TOPSIS

Step 1. Determining sets of alternatives, criteria, linguistic variables and decision-makers.

As usual with MCDM methods, the alternatives to be ranked, the criteria to be used in the ratings and the group of decision-makers are determined. In view of this, let $A = \{A_1, A_2, \ldots, A_m\}$be the set of alternatives to be considered, $A = \{C_1, C_2, \ldots, C_n\}$, the set of criteria and, $k = \{D_1, D_2, \ldots, D_d\}$ the sets of decision makers. Equation. (41), shows a decision matrix for decision maker, $k = 1,2, \ldots, d$

$$\tilde{k} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array}, i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \qquad (42)$$

where $\tilde{x}_{ij}$is the rating of alternative A_i with respect to criterion C_j both expressed in intuitionistic fuzzy sets (IFS). This implies that the rating of a decision maker k, is expressed as $\tilde{x}_{ij}^k = \langle \tilde{\mu}_{ij}^k, \tilde{v}_{ij}^k, \tilde{\pi}_{ij}^k \rangle$. Additionally, linguistic variables (criteria) to be used in the assessment and subsequent ranking are determined. The linguistic variables (criteria) are then expressed in linguistic terms and used to rate each linguistic variable. The linguistic terms are further transformed into intuitionistic fuzzy numbers (IFNs). Linguistic terms are qualitative words that reflect the subjective view of an expert or decision maker about the criteria per each alternative under consideration [137]. The various linguistic variables, terms as well as the IFNs are expressed on a scale of 0-1 and demonstrated in the case study section.

Step 2. Determining importance weights of decision-makers

In this step, the weights of decision makers, if relevant to the study, are determined based on their relative importance towards the final decision to be made. This is premised on the assumption that not all decision-makers are equal in importance and that there is a higher decision authority that rates to assign weight to the decision makers. It must be noted that not all decision problems have the decision makers weighted. In this study, because of the peculiarity of merging two decision points, the decision makers (small group of experts) are considered not to have the same importance. The ratings of the decision makers are expressed linguistically in intuitionistic fuzzy number (IFN) format. Let $\widetilde{D}_k = \langle \tilde{\mu}_k, \tilde{v}_k, \tilde{\pi}_k \rangle$be an intuitionistic fuzzy number expressing the rating of a *k*th decision maker. Then the importance weight of the *k*th decision maker may be expressed as in Eq. (43) below and cited in [20].

$$\tilde{\rho}_k = \frac{\left(\tilde{\mu}_k + \tilde{\pi}_k\left(\frac{\tilde{\mu}_k}{\tilde{\mu}_k + \tilde{v}_k}\right)\right)}{\sum_{k=1}^{d}\left(\tilde{\mu}_k + \tilde{\pi}_k\left(\frac{\tilde{\mu}_k}{\tilde{\mu}_k + \tilde{v}_k}\right)\right)} \tag{43}$$

Step 3. Determining weights of each criterion

In this step, the importance or the weight of each criterion are determined. However, peculiar to this hybrid framework, the weights of the criteria are determined through the preferences and trade-offs of the large decision group, using conjoint analysis. In Eqn. (44), W_j denotes the weight of the criterion C_j representing the importance that decision makers (large group) assign to each of the attributes (criteria) in the conjoint analysis modelling. The parameters in Eqn. (44) are explained in Eqn. (41).

$$W_j = \frac{Max(\beta_{ij}) - Min(\beta_{ij})}{\sum_{j=1}^{t}[Max(\beta_{ij}) - Min(\beta_{ij})]} \times 100 \tag{44}$$

Step 4. Aggregation of decisions

In step 4, the ratings of the decision makers concerning the alternatives and criteria importance which are expressed in intuitionistic fuzzy sets are aggregated. Let $\tilde{s}^k = \left(\tilde{x}_{ij}^k\right)_{m\times n}$ express the intuitionistic fuzzy matrix of each of the decision makers and $\tilde{\rho} = [\tilde{\rho}_1, \tilde{\rho}_1, \dots, \tilde{\rho}_d]$, the importance weight of each decision maker where $\sum_{k=1}^{d} \tilde{\rho}_k = 1, \tilde{\rho}_k \in [0,1]$.

The importance of aggregation in group decision making processes cannot be overemphasized. Aggregation operators are used to sum up all individual ratings into a composite decision for the group of decision makers. In fuzzy decision modelling, many aggregation operators have been proposed with the majority belonging to the families of averaging operators, ordered weight aggregation (OWA), compensatory operators, geometric operators, Shapley averaging operators, Sugeno integrals, Choquet operators and many other hybrid forms. In this paper, the Intuitionistic Fuzzy Weighted Averaging (IFWA) operator is used to aggregate decision makers' (small group) preferences for the alternative ratings. The (IFWA) operator by Xu [36] is preferred because it is simpler, yet efficient [35]. In Eq. (45) is the IFWA operator where $\tilde{S}_{ij}$is the aggregated decision matrix.

$$\tilde{S}_{ij} = IFWA_p\left(\tilde{s}_{ij}^{(1)}, \tilde{s}_{ij}^{(2)}, \dots, \tilde{s}_{ij}^{(d)}\right) = \tilde{\rho}_1\tilde{s}_{ij}^{(1)} + \tilde{\rho}_2\tilde{s}_{ij}^{(2)} + \dots + \tilde{\rho}_{\mathrm{d}}\tilde{s}_{ij}^{(d)}$$

$$=\langle 1 - \prod_{k=1}^{d}\left(1 - \tilde{\mu}_{ij}^{(k)}\right)^{\tilde{\rho}_{\mathrm{k}}} \prod_{k=1}^{d} \nu^{\tilde{\rho}_{\mathrm{k}}}\rangle \tag{45}$$

Step 5. Constructing weighted aggregation of intuitionistic fuzzy sets

The next step computes the aggregated weighted intuitionistic fuzzy set by multiplying the weight vector of the criteria set by the aggregated decision matrix obtained in step 4. The weighted decision matrix is expressed in Eq. (46) below.

$$W \times S = \widetilde{W}^T \times \langle \tilde{\mu}_{ij}^k, \tilde{v}_{ij}^k \rangle = \langle \tilde{\mu}_{ij}^k, \tilde{v}_{ij}^k \rangle \tag{46}$$

Step 6. Determining intuitionistic fuzzy positive A^+ *and negative* A^- *ideal solutions*

At this stage, the criteria are separated into a so-called benefit and cost criteria. Let B and C respectively represent the benefit and cost criteria. Then A^-which maximizes the cost criteria while minimizing benefit criteria, and A^+which maximizes the benefit criteria and minimizes cost criteria are computed as follows:

$$A^+ = \left(\mu_{A^+W}(x_j), v_{A^+W}(x_j)\right) A^- = \left(\mu_{A^-W}(x_j), v_{A^-W}(x_j)\right) \tag{47}$$

where

$$\mu_{A^+W}(x_j) = \left(\left(\max_i \mu_{A_i.W}\,(x_j) |\, j \in B\right), \left(\min_i \mu_{A_i.W}\,(x_j) |\, j \in C\right)\right) \tag{48}$$

$$v_{A^+W}(x_j) = \left(\left(\min_i v_{A_i.W}\,(x_j) |\, j \in B\right), \left(\max_i v_{A_i.W}\,(x_j) |\, j \in C\right)\right) \tag{49}$$

$$\mu_{A^-W}(x_j) = \left(\left(\min_i \mu_{A_i.W}\,(x_j) |\, j \in B\right), \left(\max_i \mu_{A_i.W}\,(x_j) |\, j \in C\right)\right) \tag{50}$$

$$v_{A^-W}(x_j) = \left(\left(\max_i v_{A_i.W}\,(x_j) |\, j \in B\right), \left(\min_i v_{A_i.W}\,(x_j) |\, j \in C\right)\right) \tag{51}$$

Step 7. Computing separating measures

The distances $d_{IFS}^+$and d_{IFS}^-, which express the distances of each alternative from A^+ and $A^-$are calculated as shown in Eq. (52) and (53) respectively. These distances are computed as intuitionistic sets.

$$d_{IFS}^+(A_iA^+) = \sqrt{\sum_{j=1}^m \left[\left(\mu_{A_iW}(x_j) - \mu_{A^+}(x_j)\right)^2 + \left(v_{A_iW}(x_j) - v_{A^+}(x_j)\right)^2 + \left(\pi_{A_iW}(x_j) - \pi_{A^+}(x_j)\right)^2\right]} \tag{52}$$

$$d_{IFS}^-(A_iA^-) = \sqrt{\sum_{j=1}^m \left[\left(\mu_{A_iW}(x_j) - \mu_{A^-}(x_j)\right)^2 + \left(v_{A_iW}(x_j) - v_{A^-}(x_j)\right)^2 + \left(\pi_{A_iW}(x_j) - \pi_{A^-}(x_j)\right)^2\right]} \tag{53}$$

Step 8. Computing relative closeness coefficient and ranking of alternatives

The relative closeness coefficient also known as relative gaps degree *CCi*, is used to determine the ranking of the *i*th alternative. This is computed using Eq. (54) below:

$$CC_i = \frac{d_{IFS}^-(A_iA^-)}{d_{IFS}^-(A_iA^-) + d_{IFS}^+(A_iA^+)} \tag{54}$$

The highest value of CCi determines the best alternative implying that the chosen alternative is concurrently closer to A+ and farther away from A-

- **NUMERICAL EXAMPLES**

9 INTRODUCTION TO NUMERICAL EXAMPLES

This section presents 3 numerical examples to demonstrate the applicability of the proposed conjoint analysis – intuitionistic fuzzy TOPSIS framework for merging two-tier decision points into a composite one. In each of the three examples (cases), preferences of a lower decision making body (large group) is incorporated into preferences of a higher decision making body (small group or experts). As mentioned earlier, the conjoint analysis method models the preferences of a large decision group into criteria weights and incorporates the weights into another decision level by a relatively small group involved in ranking and selection of competing sets of alternatives. The aggregate conjoint analysis model was used to generate the part-worth utilities of each attribute (criteria). Furthermore, the MONANOVA (Monotone Analysis of Variance) estimation was used in producing the part-worths utilities at each level of attributes. The second part involved the use of intuitionistic fuzzy TOPSIS in the ranking and selection of various competing alternatives demonstrated in 3 numerical examples. Each numerical example captures the 'special' scenario where such 2-tier decision making model would be relevant in an ideal world situation. In each example, the decision problem is formulated in a story-like form to demonstrate the need to apply the proposed hybrid MCDM model.

The first numerical example looked at an ideal situation of building compromises between shareholders and board management group in a micro finance organization. Preferences of shareholders regarding the selection of a new manager are converted into relative criteria weights using conjoint analysis and subsequently, candidates for the position of a new manager are ranked and selected with intuitionistic fuzzy TOPSIS.

In the second numerical example, customer preferences regarding a company's evaluation of its third party distributing agents (distributors) are modelled with the proposed hybrid method in a two-tier structure. The demonstration again shows how the hybrid framework incorporates customer concerns into management selection decisions.

The third numerical example looks at a hypothetical situation of a company selecting a recruitment Process Outsourcing Vendor as a partner. To this, company management team samples preferences of other managers in the sector regarding the characteristics they wish to see in a recruitment outsourcing vendor along with the trade-offs they would wish to make. Similarly, these preferences are modelled with conjoint analysis into criteria weights and subsequently, intuitionistic fuzzy TOPSIS helps in the selection stage. The following sections present the numerical examples.

9.1 NUMERICAL EXAMPLE 1.

(Incorporating Shareholder Preferences into Board Management Selection Decisions)

9.1.1 Background to the problem

Most decisions in the corporate world are traditionally taken by a team of experts from fields and disciplines that matter to the problem [138]. These team of experts often constitute the board of management [139] or simply company directors. In recent times, the practice of heavily relying on board of management for critical organizational decisions such as (1) selection of a new manager/director, (2) sale of assets, (2) determining performance-based compensations and (3) amount of money to distribute among shareholders among others [140], have been challenged by legal representatives of shareholders as they seek more for ‘powers’. In particular with recent corporate governance failures at HIH Insurance Limited [141, 142], Enron Inc. [143, 144] and WorldCom Inc [145,144], shareholders around the world have been empowered to seek for increased participation in critical decisions management decisions ([140, 146]. Shareholder activism and its related empowerment agenda for an increased decisional roles have gained momentum across the globe especially in America, Japan, United Kingdom, South Korea, and Australia [147]. With some high profile cases such as Carl Icahn versus Time Warner, Kirk Kerkorian versus General Motors and Nelson Peltz versus H.J. Heinz & Co. [148], shareholders continue to seek an increase participatory role in corporate governance by demanding for more decisional powers.

However, with such momentum for a greater representation of shareholders in critical decisions, the challenge has been identifying those exact situations where such collaborative effort between the board of management (company directors) and shareholders can be reached. This numerical example uses the proposed 2-tier conjoint analysis-intuitionistic fuzzy TOPSIS hybrid decision making model to present a real-world example of how such management board – shareholder decision making collaboration can be explored. The case study demonstrates how certain powers can be ceded to shareholders in an organizational setting. In the numerical example of selecting a new manager in a microfinance organization, shareholders who belong to a ‘large group’ of decision makers are given the powers to determine the relative importance of the criteria to be used in the manager selection process. Their preferences and trade-offs are modelled into criteria (attributes) weights and subsequently used by board of management (small group) in the selection. Such shared decision making model has the potential of deepening trust and accountability between shareholders and management board.

9.1.2 The Decision Problem

The problem is to select a new manager to replace an outgoing one. Given recent history of shareholder activism for greater representation in key decisions, management has agreed to involve shareholders by incorporating their preferences in terms of the importance they attach to the criteria to be used in the selection of a new manager. To carry out this, a number of problems were identified. The first was finding an appropriate method to model the preferences of 126 shareholders and second, how to incorporate their decisions in terms of criteria weights into the decision making process. Conjoint analysis (CA) and intuitionistic fuzzy TOPSIS method were chosen they mirror the human rationale in selection. In the following section, the phases for the hybrid framework are outlined. CA in particular was ideal for modelling large preference data

Phase 1: *Selecting Attributes and Levels for modelling with Conjoint Analysis*

The attributes (criteria) for selecting a new manager for the microfinance company, was primarily based on expert advice and past literature on manager selection problems. Based on these two pre-requisite, five key attributes or criteria were identified as relevant for the problem. These were Knowledge of Business, Educational Background, Relevant Experience, Management skills and Soft Skills. All the 5 attributes were assigned with three levels as shown in Table 9. A fractionated factorial design of 12 profiles (packages) was adopted and designed with the help of the Conjoint Design module in the XLSTAT software (2014 version). Table 8, shows the 12 different profiles presented in this study to elicit shareholder preferences based on rankings preference scale. The fractionated factorial design experiment was useful considering that a full factorial design would have yielded 243 stimuli which would have been a daunting task for the shareholders. The aggregate conjoint analysis model was employed in generating the part-worths of each attribute. Additionally the MONANOVA (Monotone Analysis of Variance) estimation method was adopted for realizing the part-worths utilities because it generates relatively better scores by combining the linear regression model with a monotonic transformation of the response variable [151].

The part-worth utilities, which are an individual respondent's (shareholder) 'value system' or importance attached to each level of each criteria, are first computed for each individual (shareholder) using Eq. (40). These are then aggregated and averaged (mean) for all decision makers (shareholders) using Eq. (41). For example, in column 7 and 8 in Table 8 are two preference ranking orders of two shareholders (abbreviated as *shdr*) regarding the value they assign to each of the 12 characteristic profiles. *Shdr* 1 and 2 assign a preference scale of 8 and 6 respectively to profile 1. These by the help of Eq. (40) computes an individual's utility which are subsequently aggregated for all individuals and averaged to represent the relative importance of each criteria as shown in column 4 of Table 9.

Table 8. Profiles of possible candidate managers evaluated by shareholders in the experimental design

Observation	Knowledge of Business	Relevant Experience	Management skills	Soft Skills	Educational Background	Shdr 1	Shdr 2
Profile1	Current employee candidate	More than 6yrs.	Meeting Management	Adaptability	Diploma with professional certification	8	6
Profile2	Non-employee candidate (not from a competitor)	1-3yrs.	Leadership and people management	Good oral communication	Bachelor's degree	4	5
Profile3	Non-employee candidate (from a competitor)	1-3yrs.	Meeting Management	Adaptability	Bachelor's degree	4	6
Profile4	Non-employee candidate (not from a competitor)	More than 6yrs.	Leadership and people management	Problem solving	Diploma with professional certification	2	0
Profile5	Non-employee candidate (not from a competitor)	More than 6yrs.	Finance Skills	Adaptability	Bachelor's degree	6	7
Profile6	Current employee candidate	More than 6yrs.	Meeting Management	Good oral communication	Master's degree	9	9
Profile7	Non-employee candidate (from a competitor)	More than 6yrs.	Leadership and people management	Adaptability	Master's degree	7	6
Profile8	Current employee candidate	1-3yrs.	Finance Skills	Problem solving	Master's degree	10	10
Profile9	Current employee candidate	4-6yrs.	Leadership and people management	Problem solving	Bachelor's degree	6	4
Profile10	Non-employee candidate (from a competitor)	4-6yrs.	Finance Skills	Good oral communication	Diploma with professional certification	5	4
Profile11	Non-employee candidate (not from a competitor)	4-6yrs.	Meeting Management	Problem solving	Master's degree	3	3
Profile12	Non-employee candidate (from a competitor)	More than 6yrs.	Meeting Management	Problem solving	Bachelor's degree	6	8

Table 9. Attributes (Criteria) and Levels with their part-worths utilities relative importance

Attribute (criteria)	Levels	Part-worth utilities (Mean)	Relative importance (Weight %)
C_1:Knowledge of Organization	Current employee candidate (C_{11}) Non-employee candidate (from a competitor) (C_{12}) Non-employee candidate (not from a competitor) (C_{13})	**2.1054** 0.0150 -2.1204	**37.1097**
C_2:Educational Background	Diploma with professional certification (C_{21}) Bachelor's degree (C_{22}) Master's degree (C_{23})	-0.4864 -0.5353 **1.0217**	**19.3987**
C_3:Relevant Experience	1-3yrs. (C_{31}) 4-6yrs. (C_{32}) More than 6yrs. (C_{33})	-0.1485 0.0150 **0.1335**	**10.6780**
C_4:Management Skills	Finance Skills (C_{41}) Leadership and people management (C_{42}) Meeting Management (C_{43})	**1.1745** -0.6351 -0.5394	**18.6570**
C_5: Soft skills	Good oral communication (C_{51}) Problem solving (C_{52}) Adaptability (C_{53})	**0.6750** -0.5438 -0.1312	**14.1567**

The most preferred levels in each criteria are those assigned with large part-worth utilities whereas the least preferred levels obtain small part-worth utilities.

Table 9 shows the part-worths and relative importance of the criteria for the selection of a new manager considering preferences of the entire shareholders (n=126). It can deduced that on the average, *knowledge of organization* (C_1) was adjudged the most important criteria (37.11%), followed by *educational background* (19.40%), *management skills* (18.66%), *soft skills* (14.15%) and *relevant experience* (10.68%). Regarding the part-worth utilities, the results in Table 9, also indicate that shareholder's most preferred candidate for the vacant manager position should be one with the following composite attribute levels; C_{11}, C_{23}, C_{33}, C_{41} and C_{51}. In effect the profile of the new manager should be one with attributes and levels such as: a current employee candidate (C_{11}) with a Master's degree (C_{23}), who has relevant experience of more than 6yrs. (C_{33}), possesses adequate financial Skills (C_{41}) and has a Good oral communication (C_{51}). In figure 16, the sizes of the boxes indicate those levels that contribute positively to shareholder preferences the characteristics of the new manager selection.

The preference modeling with conjoint analysis is effective since shareholders traded-off several characteristics to arrive at this candidate profile. Typical MCDM methods would normally not offer the chance for such clear and easily interpretable trade-offs.

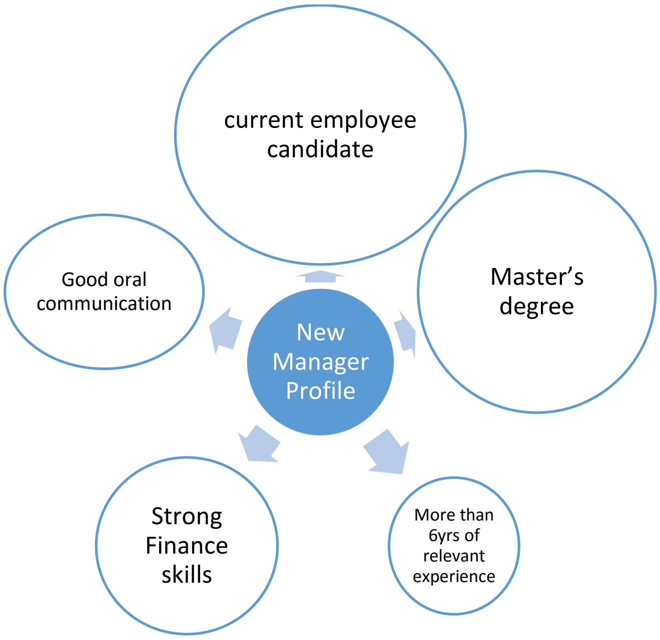


Fig. 16: Shareholder's characteristic preferences of an ideal new manager

Phase 2: *Determining importance weights of board management members*

The importance of each board management member (decision maker) in terms of the weight of their ratings are determined using Eq. 43. In Table 10, the importance weights of each of the 7 decision makers are presented. The ratings of decision maker 3 carries more weight than the others.

Table 10. Importance weights of decision makers

	D_1	D_2	D_3	D_4	D_5	D_6	D_7
Weight	0.16	0.15	0.25	0.10	0.09	0.12	0.13

Phase 3: *Determining the linguistic Variables and Terms*

The board management team evaluates the candidates based on five attributes (criteria) using linguistic terms (*Very Low*, *Low*, *Medium*, *High* or *Very high*) to rate each of the competing candidates. Table 11 below shows the linguistic terms with their assigned IFNs.

Table 11. Linguistic scales for candidates' ratings

Linguistic terms	**IFS fuzzy number**	**Ratings of Alternatives**
Very Low (VL)	$\langle 0.05,0.95 \rangle$	Very Low (VL)
Low (L)	$\langle 0.2,0.75 \rangle$	Low (L)
Medium(M)	$\langle 0.55,0.4 \rangle$	Medium(M)
High (H)	$\langle 0.75,0.2 \rangle$	High (H)
Very High (VH)	$\langle 0.95,0.05 \rangle$	Very High (VH)

Phase 4: *Constructing and aggregating the fuzzy decision matrix*

In this phase, the candidate ratings shown in Table 12 are aggregated using the IFWA operator as expressed in Eq. (45) under the methodological steps of intuitionistic fuzzy TOPSIS. The ratings represent the opinions of each of the 7 board members (DMs) regarding the suitability of each candidate for the position of a new manager.

Table 12. Decision makers' ratings of competing candidates

		D_1	D_2	D_3	D_4	D_5	D_6	D_7
C_1	A_1	L	M	L	L	M	VL	M
	A_2	M	M	M	M	M	M	M
	A_3	VH	H	VH	VH	VH	VH	VH
	A_4	H	H	H	M	H	M	VH
	A_5	VL	M	L	L	M	VL	L
C_2	A_1	M	M	L	L	M	H	M
	A_2	H	M	H	M	H	M	H
	A_3	VH	H	H	VH	VH	VH	VH
	A_4	VH	H	H	H	H	H	H
	A_5	M	M	M	L	L	M	L
C_3	A_1	L	VL	VL	M	M	VL	H
	A_2	L	H	M	M	M	M	M
	A_3	H	H	H	M	VH	H	VH
	A_4	VH	M	VH	M	M	M	M
	A_5	M	M	H	H	H	M	H
C_4	A_1	M	M	VL	L	M	M	L
	A_2	L	L	H	L	H	L	L
	A_3	VH	VH	VH	VH	VH	H	VH
	A_4	VH	H	VH	H	M	M	M
	A_5	H	M	M	M	H	M	L
C_5	A_1	M	M	M	M	M	VH	M
	A_2	M	L	H	M	H	H	H
	A_3	H	H	H	H	H	VH	VH
	A_4	VH	H	VH	H	VH	H	VH
	A_5	M	L	L	M	L	M	L

Phase 5: *Constructing weighted aggregated intuitionistic fuzzy matrix*

The aggregated decisions are weighted at this phase using the weights obtained from shareholders through the use of the conjoint analysis in Table 9. The weighted aggregated fuzzy matrix is realized using Eq. (46) where W_j represents the importance shareholders attach to the attributes and S_{ij} is the normalized aggregated judgements from the board of management. The results of the weighted fuzzy decision matrix are as shown in Table 13 below.

Table 13. Aggregated weighted intuitionistic fuzzy decision matrix (A^+,A^-)

	C1	C2	C3	C4	C5
A1	(0.131, 0.745, 0.124)	(0.094, 0.564, 0.342)	(0.035,0.673, 0.292)	(0.071, 0.653, 0.276)	(0.093, 0.409, 0.498)
A2	(0.204, 0.623, 0.173)	(0.134, 0.402, 0.464)	(0.059, 0.463, 0.479)	(0.086, 0.576, 0.338)	(0.092, 0.392, 0.515)
A3	(0.879, 0.117, 0.004)	(0.72, 0.24, 0.039)	(0.386, 0.555, 0.059)	(0.627, 0.323, 0.051)	(0.531, 0.408, 0.061)
A4	(0.722, 0.242, 0.036)	(0.642, 0.301, 0.057)	(0.373, 0.574, 0.054)	(0.562, 0.383, 0.055)	(0.58, 0.368, 0.052)
A5	(0.10, 0.804, 0.096)	(0.089, 0.588, 0.323)	(0.072, 0.347, 0.58)	(0.101, 0.518, 0.381)	(0.051, 0.649, 0.301)

It can be seen from the weighted aggregated intuitionistic fuzzy decision matrix in Table 13 that, decision makers seem very satisfied with alternative A3, followed by A4 since most of their membership functions according to Eq. 29 are relatively higher than their non-membership and hesitancy degrees.

Phase 6: *Determining distance of each alternative from IFPIS and IFNIS*

The intuitionistic fuzzy positive-ideal solution (FPIS) and the intuitionistic fuzzy negative-ideal solution (FNIS), are defined respectively as A^+ and A^-. At this stage, the criteria are separated into a so-called benefit and cost criteria. Respectively, A^- maximizes the cost criteria and minimizes the benefit criteria whiles A^+ maximizes the benefit criteria and minimizes the cost criteria. The separation is done using equations 47-51.

$$A^+ = [(1,1,1), (1,1,1), (1,1,1), (1,1,1), (1,1,1), (1,1,1)] \quad (55)$$

$$A^- = [(0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,0)] \quad (56)$$

Phase 7: *Computing separating measures*

The distance measures of $d^+_{IFS}$and d^-_{IFS} are computed from each alternative to the fuzzy positive and negative ideal solutions using Eqns. (52) and (53) respectively. Subsequently, this results in Table 14 below. Additionally, the closeness coefficient that ultimately determines the ranking order of the alternatives are calculated using Eq. (54). It can be seen that per the numerical example, alternative 3 (A3) happens to be the best candidate for the vacant managerial position followed by A4, A2, A5 and A1 in that order.

Table 14. Relative closeness coefficient and ranking

	D^+	D^-	CCi	Ranking of Alternatives
A1	1.023	0.104	0.24	5
A2	0.441	0.309	0.46	3
A3	0.004	1.303	0.95	1
A4	0.026	1.088	0.87	2
A5	0.886	0.22	0.33	4

The result of the distance measurement is shown in table 14 together with their relative closeness coefficient (CCi) and the resulting rank of each alternative (candidates). Table 14 shows how the 2-tier decision making structure that incorporates shareholder views into board management decisions settled on candidate A3 as the best alternative.

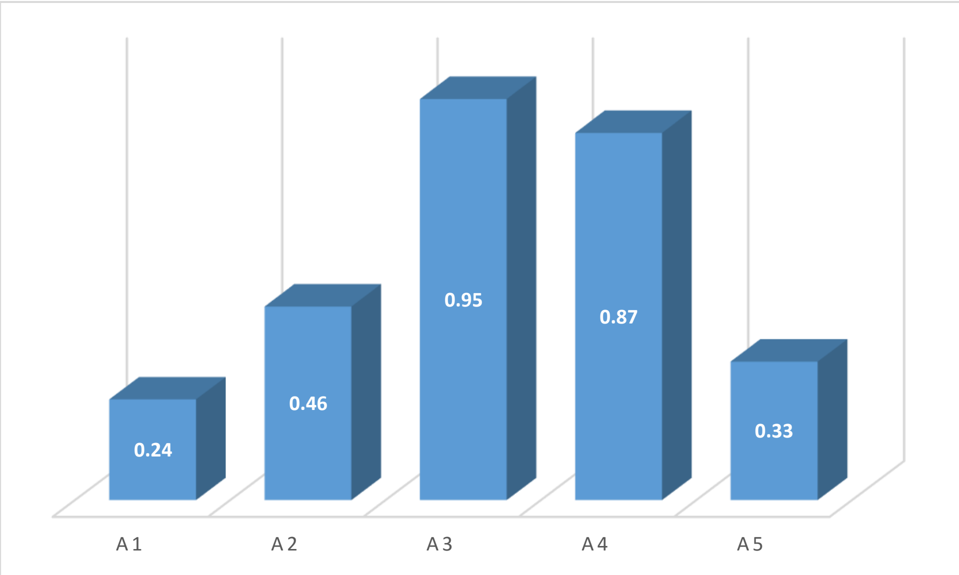


Fig. 17: Ranking of alternatives (candidates)

9.2 NUMERICAL EXAMPLE 2

(Leveraging Customer Preferences in Company Distributor Selection Decisions)

9.2.1 Background to the problem

Today, multinational companies are taking advantage of the massive and easily generated customer preferential data to tailor products and services to the needs of their clients. Customer preferential data modelling goes beyond the basic demographic details of who patronizes a product or service. In view of this, the ability to anticipate customer needs is often viewed as a strong step in building a strong sustainable customer loyalty profile for a company [149]. Whiles customer preference modelling has been an active research in marketing and consumer studies, there is little in terms of models that effectively integrate customer opinions right into management decisions. In a rapid changing business environment where opinions and preferences of the customer take centre stage, the ability to integrate customer decisions into management key decisions is welcoming.

Decision models based on customer utility maximization especially those expressed as weighted additive models have dominated research in consumer studies [150]. One of such decision tools is conjoint analysis. As mentioned in chapter 8, both conjoint analysis and MCDM methods share some common characteristics especially in their use of linear or weighted additive models to extract utilities. However, they have rarely been deployed together. In this numerical example, a demonstration of how the two methods can be used together, is explored in another 2-tier decision making model. In this model, customer preferences are factored into company distributor selection problem. Similar to the first numerical example, the idea is to convert the preferences and trade-offs of a large group of decision makers (in this case, customers) into criteria or attribute weights and subsequently incorporate into key management decisions. This novel type of decision making mirrors real world situations of accommodating decisions of a lower decision making body into a higher one. In the decision problem described below, a company solicits customer opinions in the form of an evaluation about the company's distributors who regularly interact with customers (clients). Manufacturing distributors serve as the link between manufacturers (of product or service) on one hand and their customers.

To retain distributors who meet the company's requirements expressed in several criteria, customer preferences are modelled into criteria weights and subsequently factored into another level of decision making where management executives of the company select those to retain and those to relieve off contract. Using conjoint analysis, the company is able to estimate the relative importance their customers attach to the criteria for distributor selection and the extent of trade-offs they are willing to make.

The numerical example again deploys the proposed 2-tier *conjoint analysis-intuitionistic fuzzy TOPSIS* hybrid decision making model of how such customer-management relationship in decision making can be effectively utilized. Such shared decision making scenario builds trust between customers and company management whiles again revealing customer preferences for the company to tailor the right solutions to them.

9.2.2 The Decision Problem

Management executives in a manufacturing company periodically evaluate the performance of their distributors so as to decide between those to retain and those to relieve off contract. However, since the distributors deal directly with customers, the company finds it expedient to involve customer opinions, preferences and trade-offs in the selection, further retention and relieving of their distributors. It anticipated that customers in making their choices in the evaluation would make trade-offs. In all, 300 customer preferences and trade-offs are modelled using conjoint analysis in the study. Conjoint analysis is found ideal because of its ability to model trade-offs and handle large preferential inputs. Intuitionistic fuzzy TOPSIS is subsequently used the purchasing and supply executives in the ranking and selection stage. In the following section, the phases for the hybrid framework are outlined.

Phase 1: *Selecting Attributes and Levels for modelling with Conjoint Analysis*

Preliminary literature study and expert opinions settled on 7 criteria (attributes) as relevant for the study. These are: service quality, on-time delivery, cost, reputation, and responsiveness to customer needs, State of transportation and relationship with distributing company.

In Table 15, the relevant attributes (criteria) and their levels are shown. Again, a fractionated factorial design of 15 profiles (packages) were opted to make it easier and quick for responses. Table 16, shows these 15 different profiles used in the customer preferences modelling of company distributors. The fractionated factorial design experiment was used. The aggregate conjoint analysis model was employed in generating the part-worths of each attribute. Further, the MONANOVA (Monotone Analysis of Variance) estimation method was adopted for estimating the part-worths utilities [151]. In Table 15 are the attributes and their respective levels, the part-worth utilities for each level and the relative importance (weights) of each attribute (criteria) attained in the numerical example.

Table 15. Attributes (Criteria) and Levels with their part-worths utilities relative importance

Attribute (criteria)	Levels	Part-worth utilities (Mean)	Relative importance (Weight %)
C_1: Service Quality	Excellent (C_{11}) Acceptable (C_{12}) Poor (C_{13})	-0.4430 **0.2933** 0.1498	**15.0761**
C_2: On-time delivery	100% on time (C_{21}) 60% on time (C_{22}) 40% or below on time (C_{23})	**0.5512** -0.1873 -0.3639	**14.8845**
C_3: Cost	High (C_{31}) Moderate. (C_{32}) Low. (C_{33})	0.6711 **1.2860** -1.9572	**19.3060**
C_4: Reputation	Highly professional (C_{41}) Not so professional (C_{42})	**0.0543** -0.0543	**5.2598**
C_5: Responsiveness to customer needs	Highly responsive (C_{51}) Moderately responsive (C_{52}) Low level of responsiveness (C_{53})	-0.2496 -0.2309 **0.4805**	**15.7016**
C_6:State of transportation	Good (C_{61}) Poor (C_{62})	**-0.0686** -1.0561	**13.6110**
C_7: relationship with distributor	Excellent (C_{71}) Good (C_{72}) Poor (C_{73})	1.1214 **1.1247** -1.1214	**16.1609**

The most visible strength of the conjoint analysis technique especially as used in this special case of merging different levels of decisions, is its ability to explicitly show where respondents made trade-offs as well as the attributes (criteria) they prefer most in terms of their relative importance to the overall selection problem. In Table 15, the numerical example shows that customers' preferred characteristics of an ideal distributor should be one with the following composite attributes shown in figure 18.

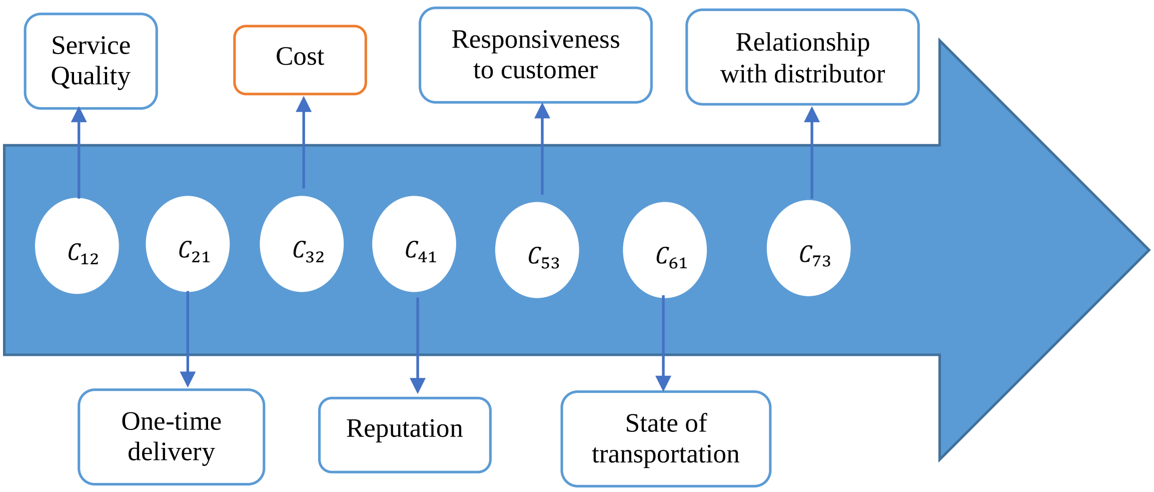


Fig. 18: Composite attributes of a preferred distributor

Table 16. Profiles of preferred distributorship characteristics sought by customers in the experimental design

Observation	Service Quality	On-time Delivery	Cost	Reputation	Customer Needs	State of transportation	Relationship with distributor
Profile 1	Acceptable	100% on time	Low	Highly professional	Low level of response	Poor	Good
Profile 2	Poor	100% on time	High	Highly professional	Low level of response	Poor	Poor
Profile 3	Acceptable	40% or below on time	Low	Not so professional	Moderately responsive	Poor	Excellent
Profile 4	Acceptable	40% or below on time	Moderate	Highly professional	Low level of response	Good	Excellent
Profile 5	Poor	40% or below on time	High	Not so professional	Highly responsive	Poor	Good
Profile 6	Excellent	100% on time	Low	Highly professional	Highly responsive	Good	Poor
Profile 7	Poor	60% on time	Low	Not so professional	Low level of response	Good	Poor
Profile 8	Excellent	60% on time	High	Not so professional	Low level of response	Poor	Excellent
Profile 9	Excellent	40% or below on time	Moderate	Not so professional	Low level of response	Good	Good
Profile 10	Acceptable	60% on time	Moderate	Not so professional	Moderately responsive	Poor	Poor
Profile 11	Acceptable	60% on time	High	Highly professional	Moderately responsive	Good	Good
Profile 12	Excellent	60% on time	Moderate	Highly professional	Highly responsive	Poor	Excellent
Profile 13	Acceptable	100% on time	High	Not so professional	Highly responsive	Good	Poor
Profile 14	Poor	100% on time	Moderate	Not so professional	Moderately responsive	Good	Excellent
Profile 15	Excellent	40% or below on time	High	Highly professional	Moderately responsive	Poor	Poor

In figure 18, the preferred profile of a distributor as judged by customers is one with an acceptable quality of service (C_{12}), 100% delivery time (C_{21}), moderate level of cost (C_{32}), who are highly professional (C_{41}), with a low level of responsiveness (C_{53}), good transportation and a good relationship with the distributor. It can further be seen that customers in their choices, traded-off responsiveness to customer needs and reputation of the distributor for a moderate cost (fees). This means customers would want management in their evaluation and selection of distributors to place more emphasis on the cost of delivery of products by distributors more than any other criteria. The weights assigned to the other criteria can be seen in Table 15 where reputation is the least considered criteria. In terms of partial utilities for each level of each attributes, moderate cost is the most preferred by customers. These derived weights are further incorporated into the evaluation, ranking and selection of the distributors and further used in deciding which distribution company to retain or relieve.

Phase 2: *Determining importance weights of decision makers*

This stage assigns weights of importance to each of the decision makers assuming that not all the decision makers are equal in importance. Using Eq. 43, the importance weights of each of the 5 decision makers are presented in Table 17. The judgements of decision maker 1 is the most important in terms of the final composite decision

Table 17. Importance weights of decision makers

	D_1	D_2	D_3	D_4	D_5
Weight	0.301	0.243	0.211	0.125	0.12

Phase 3: *Determining the linguistic Variables and Terms*

The numerical example uses 5 alternatives, 7 criteria and 5 decision makers. The third party company distributors are evaluated using linguistic terms (Very *Low, Low, Medium, High* or *Very high*) to rate their performances. Table 18 below shows the linguistic terms with their assigned IFNs.

Table 18. Linguistic scales used in the ratings

Linguistic terms	IFS fuzzy number	Ratings of Alternatives
Very Low (VL)	$\langle 0.05,0.95 \rangle$	Very Low (VL)
Low (L)	$\langle 0.2,0.75 \rangle$	Low (L)
Medium(M)	$\langle 0.55,0.4 \rangle$	Medium(M)
High (H)	$\langle 0.75,0.2 \rangle$	High (H)
Very High (VH)	$\langle 0.95,0.05 \rangle$	Very High (VH)

Phase 4: *Constructing and aggregating the intuitionistic fuzzy decision matrix*

The intuitionistic fuzzy weighted averaging operator (IFWA) expressed in Eq. (45), is used to aggregate the ratings in Table 19. The ratings represent the judgements of each of the 5 DMs regarding the performance of each candidate company distributor in the year under review.

Phase 5*: Constructing weighted aggregation of intuitionistic fuzzy matrix*

The weights obtained from the customers regarding their preferences of characteristics they cherish in a distributing company, are multiplied by the aggregated intuitionistic fuzzy matrix as expressed in Eq. (46). The results of the weighted intuitionistic fuzzy decision matrix are as shown in Table 20 below.

Table 19. Decision makers' ratings of alternatives

Criteria	Alternatives			Decision-Makers		
		D_1	D_2	D_3	D_4	D_5
C_1	A_1	L	H	L	M	H
	A_2	M	H	M	M	H
	A_3	VH	H	VH	VH	VH
	A_4	VH	VH	VH	VH	H
	A_5	H	H	H	VH	H
C_2	A_1	H	H	VL	H	M
	A_2	H	M	H	M	H
	A_3	VH	H	H	H	H
	A_4	H	H	VH	H	VH
	A_5	M	H	VH	VH	VH
C_3	A_1	L	VL	VL	M	M
	A_2	L	L	M	M	M
	A_3	M	L	L	L	M
	A_4	VH	H	VH	M	H
	A_5	M	M	H	H	M
C_4	A_1	M	M	VL	L	L
	A_2	M	L	H	H	H
	A_3	H	H	H	H	L
	A_4	H	H	VH	H	M
	A_5	VH	VH	VH	VH	H
C_5	A_1	H	L	L	L	L
	A_2	M	L	M	M	M
	A_3	M	M	M	M	M
	A_4	M	H	H	M	H
	A_5	VL	L	L	M	L
C_6	A_1	H	M	H	H	M
	A_2	H	M	H	VH	M
	A_3	VH	VH	H	H	H
	A_4	VH	VH	VH	VH	VH
	A_5	H	H	H	H	H
C_7	A_1	M	H	VL	M	M
	A_2	H	L	VL	M	H
	A_3	VH	H	H	H	H
	A_4	H	H	VH	H	VH
	A_5	M	H	VH	VH	VH

Table 20. Aggregated weighted intuitionistic fuzzy decision matrix (A^+,A^-)

	A1	A2	A3	A4	A5
C1	(0.282,0.657)	(0.351, 0.585)	(0.51,0.441)	(0.518, 0.434)	(0.439, 0.499)
C2	(0.357, 0.579)	(0.381, 0.553)	(0.468, 0.477)	(0.47, 0.475)	(0.472, 0.478)
C3	(0.188,0.771)	(0.289, 0.649)	(0.282, 0.657)	(0.664, 0.284)	(0.475, 0.451)
C4	(0.02, 0.978)	(0.03, 0.967)	(0.036, 0.962)	(0.041, 0.958)	(0.047, 0.953)
C5	(0.244, 0.698)	(0.267, 0.677)	(0.304, 0.638)	(0.375, 0.560)	(0.119, 0.846)
C6	(0.382, 0.552)	(0.413, 0.528)	(0.494, 0.455)	(0.523, 0.429)	(0.414, 0.518)
C7	(0.301, 0.641)	(0.293, 0.647)	(0.468, 0.477)	(0.47, 0.475)	(0.472, 0.478)

Phase 6: *Determining distance of each alternative from IFPIS and IFNIS*

The separation measures of distances from the intuitionistic fuzzy positive-ideal solution (FPIS) and the intuitionistic fuzzy negative-ideal solution (FNIS) are computed at this stage. Using Eqs. (52) and (53) defined respectively as A^+ and A^- , IFPIS and IFNIS are presented in Eq. 57 and 58 respectively. The elements of A^+ and A^- are used because some of the attributes (criteria) for the selection of the company distributing agents were deemed as benefits and some as costs. In determining A^+ and A^-, criteria (C1, C2, C4, C5 and C7) are considered as benefits whiles the C3 is designated as a cost criterion.

$$A^+ = \begin{bmatrix} (0.518,\ 0.434,\ 0.048),\ (0.47,\ 0.475,\ 0.055), \\ (0.188,\ 0.771,\ 0.041), (0.047,\ 0.953,\ 0.00), \\ (0.375,\ 0.560,\ 0.065), (0.523,\ 0.429,\ 0.048), \\ (0.468,\ 0.477,\ 0.055) \end{bmatrix} \tag{57}$$

$$A^- = \begin{bmatrix} (0.282, 0.657,\ 0.061), (0.357,\ 0.579,\ 0.065), \\ (0.664,\ 0.284,\ 0.053), (0.02,\ 0.978,\ 0.002), \\ (0.119,\ 0.846,\ 0.035), (0.382,\ 0.552,\ 0.066), \\ (0.293,\ 0.647,\ 0.06) \end{bmatrix} \tag{58}$$

The result of the distance measurement is shown in Table 21 together with their relative closeness coefficient (CCi) and the resulting rank of each alternative (distributing company).

Table 21. Relative closeness coefficient and ranking

	D^+	D^-	CCi	Ranking of Alternatives
A1	0.507	0.709	0.583	2
A2	0.445	0.582	0.567	3
A3	0.188	0.753	0.8	1
A4	0.682	0.612	0.473	4
A5	0.604	0.449	0.426	5

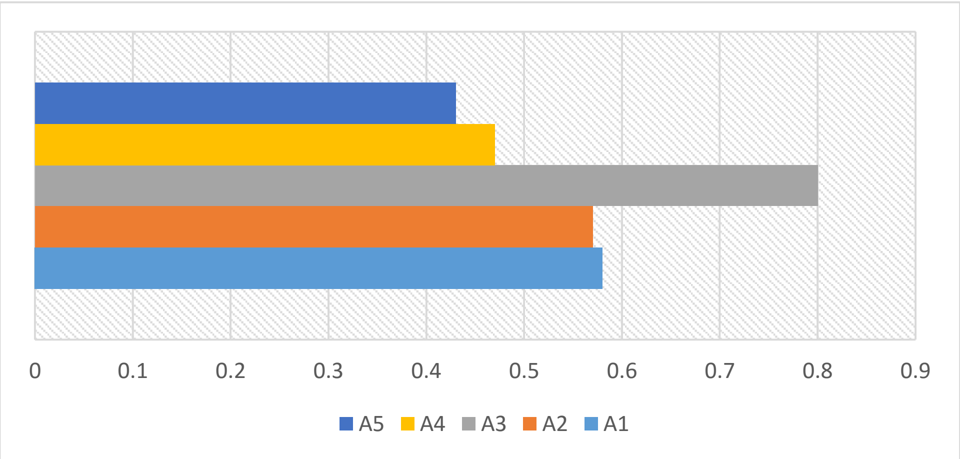


Fig. 19: Final ranking of distributing company distributor

The results in Table 21 and figure 19 show that alternative A3 is adjudged the ideal solution. The overall result shows the seamless integration of two decision points from different decision making bodies.

9.3 NUMERICAL EXAMPLE 3

(Managers' Preferences and Trade-offs in recruitment Process Outsourcing Vendor Selections)

9.3.1 Background to the problem

As multinational companies evolve, outsourcing decisions have become an integral part of key management decisions. In outsourcing, an organization decides to purchase a product or a service that hitherto was provided in-house [162] through a third party company. This process is used by many organizations to transfer recurring internal activities and decision rights to third party providers governed by a contract [163]. One of several key management decisions that organizations were reluctant to outsource was human resource recruitment. Today, there are tens of thousands of recruitment process outsourcing vendors across the globe partnering organizations in selecting and training high quality job candidates. Recruitment process outsourcing vendors, popularly known simply as recruitment agencies, play key roles in attracting, recruiting and training high caliber staff for various job positions. Since recruiting the right human resource or otherwise can have positive or negative implications for organizations, the decision to select a recruitment agency as a Human Resource (HR) partner must be handled with outmost care.

The satisfaction of an organization regarding a recruitment process outsourcing vendor can ultimately be measured by the caliber of quality job candidates they recruit. However, other relevant criteria may include the cost to the organization (fees), the time taken to hire as well as the quality of communication during recruitment process. In view of this, organizations have to have in place relevant sets of criteria to identify and select the right recruitment agency to partner in their staff recruitments. Typically, recruitment process outsourcing vendors provide their unique skills and competencies, brand name as well as their recruitment database full of contacts. Since recruitment of a right agency is key, the decision making model to select has to be thorough involving those who matter. In this numerical example, the proposed hybrid two-tier decision making model consisting of conjoint analysis and intuitionistic fuzzy TOPSIS is used to incorporate HR managers' preferences and trade-offs into management decisions about selecting a recruitment process outsourcing vendor from a list of shortlisted agencies. The numerical example again demonstrates how two levels of decision making can merged into achieving the same selection goals.

9.3.2 The Decision Problem

The problem involves selecting by ranking, candidate recruitment process outsourcing vendor to partner a Manufacturing firm in its staff recruitments. The ultimate decision to select a recruitment agency lies with the company's directors. However, to get the best of their decision, the company has decided to involve HR managers' preferences and trade-offs regarding the selection of the recruitment agency. A questionnaire designed with the help of a conjoint analysis fractionated factorial design is presented to 150 HR managers. The profiles allow the HR managers to make trade-offs in their quest to making their preferences on the right criteria characteristics mix for an ideal recruitment agency. Again conjoint analysis is preferred because of its ability to model trade-offs and handle large preferential inputs. The relative weights of criteria (attributes) derived from the preference data of the HR managers are incorporated into the final selection process by company directors. Intuitionistic fuzzy TOPSIS is utilized because of its ability to determine the satisfaction, dissatisfaction and hesitancy in a decision maker's rating and the use of distance separation to distinguish ideal solutions from worse solutions.

Phase 1: *Selecting Attributes and Levels for modelling with Conjoint Analysis*

In this numerical example, 5 criteria (attributes) are identified as relevant for the selection decision of an ideal recruitment agency. These are:

- Previous Experience in industry
- Quality of service by the agency
- Size of their database
- Recruitment service costs
- Duration of contract

In Table 22, the relevant attributes (criteria) and their levels are shown. An optimized fractionated factorial design of 12 profiles (packages) were used to elicit responses. Table 23, shows these 12 different profiles used in the HR managers' preferences regarding criteria characteristic mix of an ideal recruitment process outsourcing vendor. The aggregate conjoint analysis model was employed in generating the part-worths of each attribute. Further, the MONANOVA (Monotone Analysis of Variance) estimation method was adopted for estimating the part-worths utilities [151]. Table 22 shows the attributes (criteria), their respective levels, the part-worth utilities of each level and the relative importance (weights) of each attribute (criteria) attained in the preference modelling computation.

Table 22. Attributes (Criteria) and Levels with their part-worths utilities relative importance

Attribute (criteria)	Levels	Part-worth utilities (Mean)	Relative importance (Weight %)
C_1: Experience	5-8 yrs. (C_{11}) 3-5 yrs. (C_{12}) No experience (C_{13})	-0.1010 **0.2387** -0.1377	19.3008
C_2: Quality of service	Excellent (C_{21}) Fair (C_{22}) Poor (C_{23})	-2.0658 **1.5221** 0.5437	34.6071
C_3: Size of database	Adequate (C_{31}) Low (C_{32})	-0.1528 **0.1528**	9.0100
C_4: Recruitment service costs (fees)	High (C_{41}) Acceptable (C_{42}) Low (C_{43})	-0.2524 -0.3545 **0.6070**	22.4235
C_5: Duration of Contract	5-8 yrs. (C_{51}) 3-5 yrs. (C_{52}) 1 yr. (C_{53})	-0.2669 0.0481 **0.2188**	14.6587

Table 23. Profiles of preferred distributors' characteristics sought by customers in the experimental design

Observation	Experience	Quality of service	Size of database	Recruitment service cost	Duration of Contract
Profile 1	No experience	Poor	Adequate	Acceptable	3-5 yrs.
Profile 2	3-5 yrs.	Excellent	Low	Acceptable	3-5 yrs.
Profile 3	No experience	Fair	Adequate	Acceptable	5-8 yrs.
Profile 4	5-8 yrs.	Poor	Low	Low	5-8 yrs.
Profile 5	3-5 yrs.	Fair	Low	High	5-8 yrs.
Profile 6	5-8 yrs.	Fair	Adequate	Low	3-5 yrs.
Profile 7	5-8 yrs.	Fair	Low	Acceptable	1 yr.
Profile 8	No experience	Fair	Low	High	3-5 yrs.
Profile 9	3-5 yrs.	Poor	Adequate	High	1 yr.
Profile 10	5-8 yrs.	Excellent	Adequate	High	5-8 yrs.
Profile 11	3-5 yrs.	Fair	Adequate	Low	1 yr.
Profile 12	No experience	Excellent	Low	Low	1 yr.

In Table 22, the preferred profile for a recruitment process outsourcing vendor as adjudged by HR managers through their preferences, is one with the following composite attributes. The agency must have at least 3-5 years of experience (C_{12}), Fair level of quality of service (C_{22}), a low size of database (C_{32}), low recruitment costs (C_{43}), 1 yr. (C_{53}). The HR managers by their responses indicated that, management/directors in their evaluation and selection of a partner recruitment agency, must consider the quality of service, followed by the recruitment costs (service fees) and experience in the order of importance as seen in Table 22. The weights from the HR managers' preferences are now incorporated into the evaluation, ranking and selection of relatively best recruitment agency for the manufacturing company.

Phase 2: *Determining importance weights of decision makers*

This stage assigns weights of importance to each of the decision makers assuming that not all the decision makers are equal in importance. Using Eq. 43, the importance weights of each of the 5 decision makers are presented in Table 24. The judgements of decision maker 3 is the most important in terms of the final composite decision.

Table 24. Importance weights of decision makers

	D_1	D_2	D_3	D_4	D_5
Weight	0.25	0.25	0.3	0.12	0.08

Phase 3: *Determining the linguistic Variables and Terms*

The numerical example uses 6 alternatives, 5 criteria and 5 decision makers. The third party company distributors are evaluated using linguistic terms (*Very Low, Low, Medium, High* or *Very high*) to rate their performances. Table 25 below shows the linguistic terms with their assigned IFNs.

Table 25. Linguistic scales used in the ratings

Linguistic terms	**IFS fuzzy number**	**Ratings of Alternatives**
Very Low (VL)	$\langle 0.05,0.95\rangle$	Very Low (VL)
Low (L)	$\langle 0.2,0.75\rangle$	Low (L)
Medium(M)	$\langle 0.55,0.4\rangle$	Medium(M)
High (H)	$\langle 0.75,0.2\rangle$	High (H)
Very High (VH)	$\langle 0.95,0.05\rangle$	Very High (VH)

Phase 4: *Constructing and aggregating the intuitionistic fuzzy decision matrix*

The aggregation is carried out using intuitionistic fuzzy weighted averaging operator (IFWA) expressed in Eq. (45). In Table 27, the ratings aggregated represent the judgements of each of the 5 DMs regarding the performance of each candidate recruitment agency.

Phase 5*: Constructing weighted aggregation of intuitionistic fuzzy matrix*

At this stage, the weights derived from the preferences of HR managers regarding the characteristics they prefer to see in an ideal recruitment process outsourcing vendor, are multiplied by the aggregated intuitionistic fuzzy matrix as expressed in Eq. (46). The results of the weighted intuitionistic fuzzy decision matrix are as shown in Table 27 below.

Table 26. Decision makers' ratings of alternatives

Criteria	**Alternatives**			**Decision-Makers**		
		D_1	**D_2**	**D_3**	**D_4**	**D_5**
C_1	A_1	M	H	L	M	H
	A_2	M	H	M	M	H
	A_3	H	H	H	H	H
	A_4	H	H	H	H	H
	A_5	H	H	H	VH	H
	A_6	VH	VH	VH	H	VH
C_2	A_1	H	H	VL	H	M
	A_2	H	M	H	M	H
	A_3	M	M	M	H	H
	A_4	H	H	VH	H	VH
	A_5	M	H	VH	VH	VH
	A_6	H	VH	VH	H	M
C_3	A_1	M	VL	VL	M	M
	A_2	L	L	M	M	M
	A_3	M	L	L	L	M
	A_4	M	H	H	M	H
	A_5	M	M	H	H	M
	A_6	VH	VH	VH	VH	VH
C_4	A_1	M	M	VL	L	L
	A_2	M	L	H	H	H
	A_3	H	H	H	H	L
	A_4	H	H	H	H	M
	A_5	H	M	M	M	H
	A_6	VH	VH	VH	VH	H
C_5	A_1	H	L	L	L	L
	A_2	M	L	M	M	M
	A_3	M	M	M	M	M
	A_4	M	H	H	M	H
	A_5	M	H	L	M	M
	A_6	H	M	VH	VH	VH

Table 27. Aggregated weighted intuitionistic fuzzy decision matrix (A^+,A^-)

	A1	**A2**	**A3**	**A4**	**A5**	**A6**
C1	(0.373,0.557)	(0.42, 0.509)	(0.50,0.424)	(0.50, 0.424)	(0.53, 0.402)	(0.627, 0.323)
C2	(0.426, 0.528)	(0.687,0.553)	(0.563, 0.387)	(0.854, 0.138)	(0.858, 0.138)	(0.837, 0.152)
C3	(0.102,0.868)	(0.153, 0.803)	(0.108, 0.855)	(0.219, 0.725)	(0.221, 0.722)	(0.302, 0.648)
C4	(0.19, 0.775)	(0.565, 0.366	(0.578, 0.353)	(0.587, 0.344)	(0.501, 0.433)	(0.723, 0.238)
C5	(0.183, 0.763)	(0.251, 0.692)	(0.251, 0.692)	(0.174, 0.619)	(0.119, 0.777)	(0.422, 0.522)

In Table 27, the weighted aggregated intuitionistic fuzzy decision matrix depicts decision makers' aggregated decision to be very satisfied with alternative A6, since most of its membership functions according to Eq. 29 are relatively higher than their non-membership and hesitancy degrees. This is further reflected in the final ranking.

Phase 6: *Determining distance of each alternative from IFPIS and IFNIS*

The separation measures of distances from the intuitionistic fuzzy positive-ideal solution (FPIS) and the intuitionistic fuzzy negative-ideal solution (FNIS) are computed at this stage. Using Eqs. 52 and 53, A^+ and A^- are presented in Eq. 59 and 60 respectively. The elements of A^+ and A^- are used because some of the attributes (criteria) for the selection of the recruitment process outsourcing vendor were deemed as benefits and some as costs. In determining A^+ and A^-, criteria (C4); recruitment service costs (fees), was categorized as costs whiles the rest of the criteria are designated as benefits.

$$A^+ = \begin{bmatrix} (0.627,\ 0.323),\ (0.858,\ 0.138), \\ (0.302,\ 0.648), (0.19,\ 0.775), \\ (0.422,\ 0.522), \end{bmatrix} \tag{59}$$

$$A^- = \begin{bmatrix} (0.373, 0.557), (0.426,\ 0.528), \\ (0.102, 0.868), (0.723,\ 0.238), \\ (0.119,\ 0.777) \end{bmatrix} \tag{60}$$

The result of the distance measurement is shown in Table 28 together with their relative closeness coefficient (CCi) and the resulting rank of each alternative (distributing company). The results in Table 28 and figure 19 show that alternative A6 is adjudged the ideal solution. The overall result shows the seamless integration of two decision points from different decision making bodies.

Table 28. Relative closeness coefficient and ranking

	D+	D-	CCi	Ranking of Alternatives
A1	1.009819	0.6299274	0.38	4
A2	0.8418322	0.3972025	0.32	5
A3	0.9567217	0.3220188	0.25	6
A4	0.6210018	0.7100465	0.53	2
A5	0.7330459	0.6658741	0.48	3
A6	0.6293374	1.0216568	0.62	1

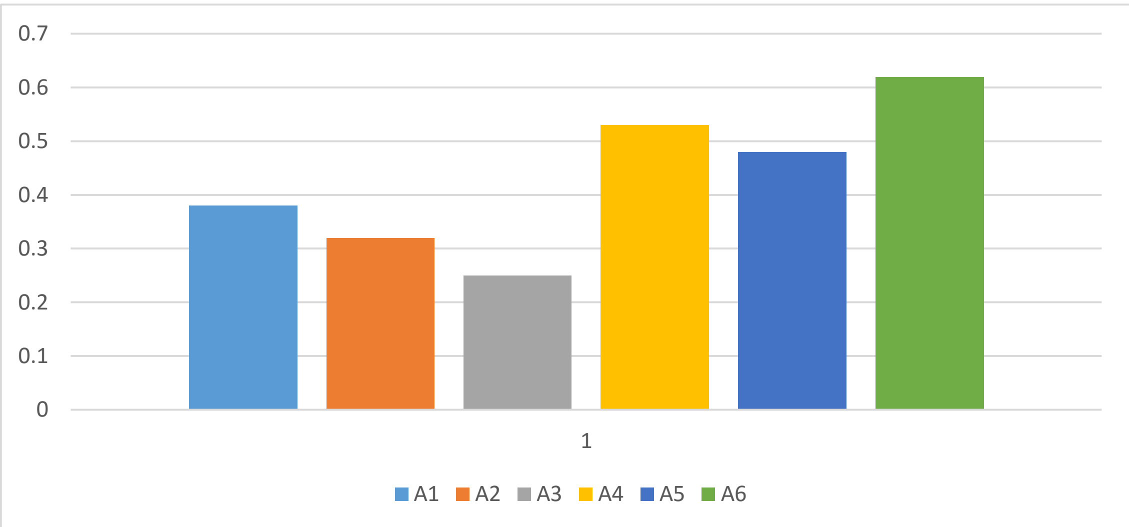


Fig. 20: Final ranking of recruitment process outsourcing vendor

- **SENSITIVITY ANALYSIS**

10 SENSITIVITY ANALYSIS IN MCDM

Sensitivity analysis plays an important role in many input-output systems. It is a technique primarily used to determine how different values of input variables will impact a particular output variable under a given set of assumptions. According to [166] sensitivity analysis is useful for a wide range of purposes including the following:

- *Decision making*: Sensitivity analysis is useful in decision making as it helps to identify critical values/parameters, to test for decision robustness and to assess the overall riskiness of decisions made.
- *Communication*: In communication, sensitivity analysis is used to enhance confidence and credibility which increases confidence. It also brings to the fore critical assumptions in a communication system.
- *Model development*: Measurement of accuracy in model development is always a requirement in scientific disciplines. Sensitivity analysis therefore helps to understand models by identifying loopholes in the system. This helps to acquire more information when relevant to improve the model.

In multi-criteria decision making, sensitivity analysis helps to test for the reliability of decisions made by decision makers by investigating the congruent effect on the final decision (ranking order) if certain key input parameters are altered. In other words, the process is invoked to determine how sensitive the overall decision is to certain alterations owing to uncertainties in the model [164]. According to [167], the presence of uncertainties in decision problems of multi-criteria nature, call for great care to be exercised when interpreting results especially in real-life applications. Literature on MCDM basically provides three kinds of approaches necessary for handling issues of variation in input parameters in a decision framework. These according to [168], [169] as cited in [167], are using a weaker information on the criteria; employing weight specification methods and or using sensitivity analyses to determine the effects on results when changes are made to the weights.

In MCDM approaches, weights are normally assigned to two things; the criteria or the decision makers. Since both are key input parameters, a change in their values can potentially change a final decision. In view of this, a sensitivity analysis model that adequately captures all scenarios of possible changes to the weights, be it criteria or decision makers, and presents possible effects on the final decision making is desirable.

This dissertation presents 3 different schemas of modelling sensitivity analysis in MCDM which are subsequently tested on the results of the 3 numerical examples used in the previous sections. Sensitivity analysis was deemed particularly necessary given the fact of incorporating two different decision points into one composite decision. Secondly, given the anticipated large number of preference data converted into criteria weights in the proposed conjoint analysis

– intuitionistic fuzzy TOPSIS framework, a use of sensitivity analysis helped to ascertain how changes to the weights assigned by the large decision making body ultimately affects the decision by the small decision body. The following 3 novel schemas are anticipated as possible cases that can alter the final decisions obtained in the numerical examples. The following are the 3 schemas used.

10.1 Schema 1: Swapping Criteria Weight

In this schema, the usual notion of anticipating criteria changes and its effect on final decision is modelled. To do this, several scenario-case based tests were conducted of possible changes to the criteria weights. These were tested on each of the 3 numerical examples. In the first scenario, the weight of the most important criteria is alternated or swapped case by case with the weights of the remaining criteria. Additionally, in each instance of swapping the weight of the most important criterion with another, the weights of all the other criteria are held constant. For example the expression, WCC_{52}, WC_1, WC_3-WC_4 means the alternation of the weights of criteria 5 and 2 whiles the weights of criteria 1, 3 and 4 are held constant. Similarly in scenario 2, the weights of the next most important criteria is swapped with the remaining criteria. Therefore the expression WCC_{24}, WC_1, WC_3, WC_5 implies interchanging of weights between criteria 2 and 4 whiles maintaining the weights of criteria 1, 3 and 5 constant. The weights swapping is conducted for the rest of the criteria in similar fashion.

Schema 1 - Numerical Example 1

In the first numerical example, the modelling of shareholder preferences resulted in the following order of importance for the attributes (criteria): $C_1 > C_2 > C_4 > C_5 > C_3$. In the first 4 cases of scenario 1 as shown in Table 29, the weight of shareholders' most important criterion (C_1: Knowledge of Organization) is alternated or swapped case by case with the weights of the remaining criteria. It must be noted that in each instance of swapping the weight of the most important criterion with another, all other criteria weights are held constant as shown in Table 29. For example in scenario 1 case 1, WCC_{12}, WC_3-WC_5 means the swapping of the weights of criteria 1 and 2 whiles the weights of criteria 3, 4 and 5 were held constant. Similarly for example, in scenario 2 case 7, the expression WCC_{25}, WC_1, WC_3-WC_4 implies interchanging of weights between criteria 2 and 5 whiles maintaining the weights of criteria 1, 3 and 4 constant. This interpretation of the scenario-case analysis is replicated all throughout Table 29.

In Table 30, the results of the sensitivity analysis based on the 10-cases in the 4-scenarios are shown along with the original ranking of the alternatives. The results show that the ranking remains unchanged in almost all the scenarios except in scenario 1 case 2 where alternative A1 outranks alternative A5 at fourth position as compared to the original ranking. However, since the ultimate aim of the decision making framework was to select the best alternative (candidate) to fill the position of a new manager, changes occurring outside the best alternative are not of utmost importance.

Table 29. Inputs for sensitivity analysis (Schema 1, Numerical example 1)

Case	**(Changes made in Wjt), J= C1, C2, …, Cn, t=1,2,...,n**	
Scenario 1	Case 1	WCC_{12}, Wc3-Wc5
	Case 2	WCC_{13}, Wc2, Wc4-Wc5
	Case 3	WCC_{14}, Wc2-Wc3, Wc5
	Case 4	WCC_{15}, Wc2-Wc4,
Scenario 2	Case 5	WCC_{23}, Wc1, Wc4-Wc5
	Case 6	WCC_{24}, Wc1, Wc3, Wc5
	Case 7	WCC_{25}, Wc1,Wc3-Wc4
Scenario 3	Case 8	WCC_{34}, Wc1-Wc2, Wc5
	Case 9	WCC_{35}, Wc2, Wc4-Wc5
Scenario 4	Case 10	WCC_{45}, Wc1-Wc3

Table 30. Results of sensitivity analysis (Schema 1, Numerical example 1)

		Scenario 1				**Scenario 2**			**Scenario 3**		**Scenario 4**
	Original Ranking	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	**5**	5	4	5	5	5	5	5	5	5	5
A2	**3**	3	3	3	3	3	3	3	3	3	3
A3	**1**	1	1	1	1	1	1	1	1	1	1
A4	**2**	2	2	2	2	2	2	2	2	2	2
A5	**4**	4	5	4	4	4	4	4	4	4	4

The sensitivity analysis therefore confirms that no matter the changes made to the weights of the criteria or the attributes, alternative A3 remains the best candidate in the selection of a new manager. It must be noted that these sensitivity analysis results may change depending on the decision problem. The novelty in the modelling is the focus in this dissertation.

Schema 1 - Numerical Example 2

The sensitivity analysis approach of swapping criteria weights is adopted also for numerical example 2. Here the resulting ranking order for the criteria as well as the alternatives are respectively; $C_3 > C_7 > C_5 > C_1 > C_2 > C_6 > C_4$ and $A_3 > A_1 > A_2 > A_2 > A_5$. The focus is to alter the weights for the criteria set to see if there are congruent effect on the alternatives ranking. In the first 6 cases of scenario 1, the weight of customers' most preferred criterion (C_3: Cost) is swapped case by case with the weights of the remaining criteria. Further, at each instance of swapping the weight of the most important criterion with another, all other criteria weights are held constant as shown in Table 31. Next the second most important criterion (C_7: relationship with distributor) is also swapped case by case with the other criteria whiles holding the rest constant. The scenario-case based sensitivity analysis is replicated all throughout in Table 31. In Table 32 and Figure 20, the results of the sensitivity analysis based on the 10-cases in the 4-scenarios are shown along with the original ranking of the alternatives. The results show that, the best 3 alternatives (A3, A1 and A4) maintain their positions in the original ranking all throughout the 10 cases. However, there are changes to the original ranking when criteria; A4 and A5 are considered.

Table 31. Inputs for sensitivity analysis (Schema 1, Numerical example 2)

Case	**(Changes made in Wjt), J= C1, C2, …, Cn, t=1,2,...,n**	
Scenario 1	Case 1	WCC_{31}, Wc2, Wc4-Wc7
	Case 2	WCC_{32}, Wc1, Wc4-Wc7
	Case 3	WCC_{34}, Wc1-Wc2, Wc5-Wc7,
	Case 4	WCC_{35}, Wc1-Wc2, Wc4, Wc6-Wc7
	Case 5	WCC_{36}, Wc1-Wc2, Wc4-Wc5, Wc7
	Case 6	WCC_{37}, Wc1-Wc2, Wc4, Wc5-Wc6
Scenario 2	Case 7	WCC_{71}, Wc2-Wc6
	Case 8	WCC_{75}, Wc1- Wc4, Wc6
Scenario 3	Case 9	WCC_{72}, Wc1, Wc3-Wc6
Scenario 4	Case 10	WCC_{51}, Wc2-Wc4, Wc6-Wc7

Table 32. Results of sensitivity analysis (Schema 1, Numerical example 2)

		Scenario 1						Scenario 2		Scenario 3	Scenario 4
	Original Ranking	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
A1	**2**	2	2	2	2	2	2	2	2	2	2
A2	**3**	3	3	3	3	3	3	3	3	3	3
A3	**1**	1	1	1	1	1	1	1	1	1	1
A4	**4**	4	4	5	4	5	5	4	4	4	5
A5	**5**	5	5	4	5	4	4	5	5	5	4

It can be seen that alternative A5 outranks alternative A4 in cases C3, C5, C6 and C10 when their positions are compared to the original ranking in column 2 of Table 32. However, since the aim of the decision making framework was to select the best alternative (distributing company), changes occurring outside the best alternative are not of utmost importance. The sensitivity analysis affirms A3 as the best distributing company in the year under review.

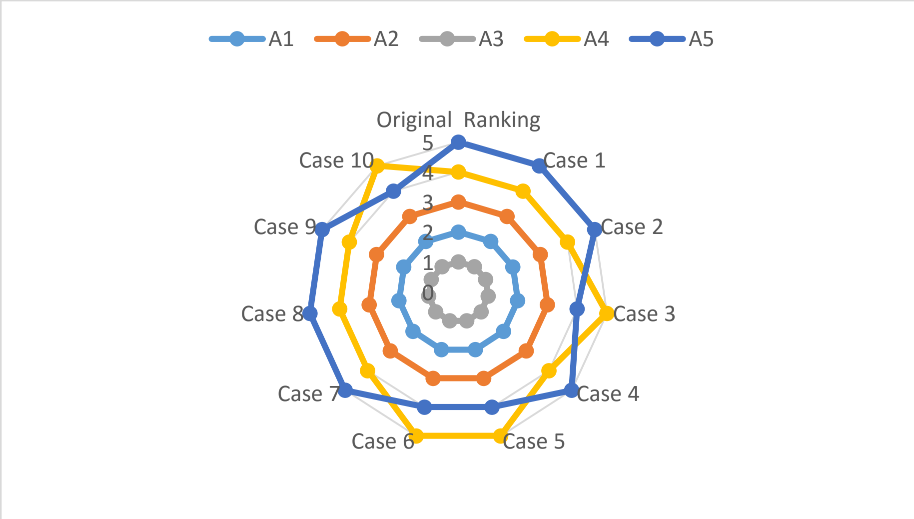


Fig. 21: Plot of sensitivity analysis (schema 1, Numerical example 2)

Schema 1 - Numerical Example 3

The order of importance of criteria set in numerical example 3 as shown in Table 22, are $C_2 > C_4 > C_1 > C_5 > C_3$. Following the weights swapping approach as demonstrated in numerical examples 1 and 2, the results of the scenario-case based sensitivity analysis is as shown in Table 34 guided by the inputs in Table 33.

Table 33. Inputs for sensitivity analysis (Schema 1, Numerical example 3)

Case	**(Changes made in Wjt), J= C1, C2, …, Cn, t=1,2,...,n**	
Scenario 1	Case 1	WCC_{21}, Wc3-Wc5
	Case 2	WCC_{23}, Wc1, Wc4-Wc5
	Case 3	WCC_{24}, Wc1, Wc3,Wc5
	Case 4	WCC_{25}, Wc1, Wc3-Wc4
Scenario 2	Case 5	WCC_{41}, Wc2-Wc3, Wc5
	Case 6	WCC_{45}, Wc1- Wc3
	Case 7	WCC_{43}, Wc1- Wc2, Wc5
Scenario 3	Case 8	WCC_{15}, Wc2-Wc4
	Case 9	WCC_{13}, Wc2, Wc4-Wc5
Scenario 4	Case 10	WCC_{35}, Wc1-Wc2, Wc4

The results show that, the best alternatives A6 is outranked by A4 in scenario 1 case 4 as shown in Table 34 and Figure 21. This means the best alternative A6, is susceptible to be been outranked when changes are made. Alternative A2 and A3 also alternate in terms of their positions in the original ranking.

Table 34. Results of sensitivity analysis (Schema 1, Numerical example 3)

		Scenario 1				Scenario 2			Scenario 3		Scenario 4
	Original Ranking	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
A1	**4**	4	4	4	4	4	4	4	4	5	4
A2	**5**	5	5	5	5	5	5	6	5	4	5
A3	**6**	6	6	6	6	6	6	5	6	6	6
A4	**2**	2	2	2	1	2	2	2	2	2	2
A5	**3**	3	3	3	3	3	3	3	3	3	3
A6	**1**	1	1	1	2	1	1	1	1	1	1

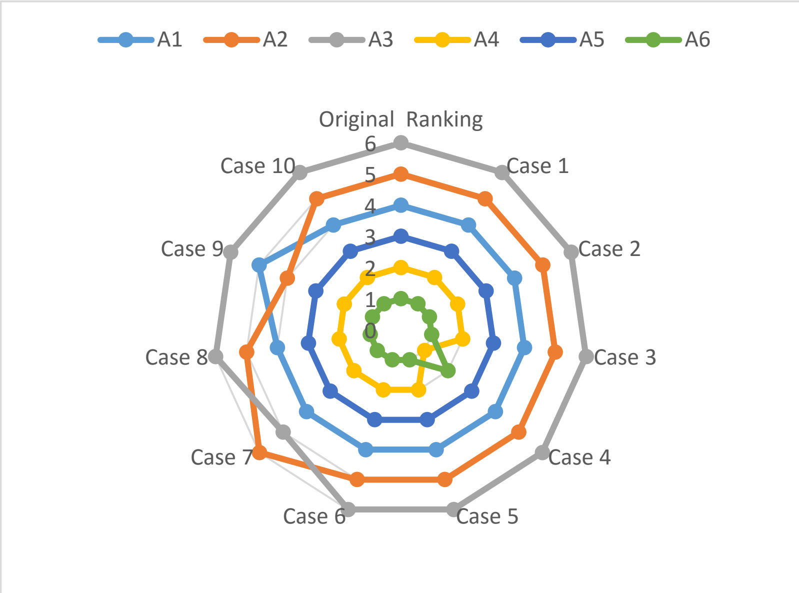


Fig. 22: Plot of sensitivity analysis (Schema 1, Numerical example 3)

10.2 Schema 2: Swapping Decision Makers' Weights

The notion of assigning weights to decision makers is not as popular as assigning weights to criteria. When used however, it is premised on the fact that not all decision makers are of equal importance. Thus the judgements of some DMs are relatively highly regarded than others. In this schema, the effects of changing DMs weights on the final decision ranking order are investigated. To do this, the scenario-case based tests were conducted where DMs weights are tweaked in 4 scenarios in 10 different cases. The investigation again uses the 3 numerical examples to ascertain the effects of decision makers' weight changes on the final ranking. In all the 3 numerical examples, weights were assigned to both DMs and criteria alike. However in this schema, just as in schema 1, when DMs weights are been tweaked, the weights of the criteria are held constant. Similar to schema 1, the first scenario under this schema alternates the weight of the most important DM in a case by case scenario with the weights of the remaining DMs. Whiles swapping the weights in each instance, the weights of all other remaining DMs are held constant. Accordingly, the expression W_{DD12}, W_{D3}-W_{D5} means the swapping of the weights of DMs 1 and 2 whiles the weights of DMs 3, 4 and 5 are held constant. This is repeated for the next most important DM with the remaining criteria in a 4 scenario 10 cases investigation. This sensitivity analysis model is applied on all the 3 numerical examples.

Schema 2 - Numerical Example 1

In numerical example 1, the order of importance of the decision makers are the following: $D_3 > D_1 > D_2 > D_7 > D_6 > D_4 > D_5$. Table 35 presents the inputs for the sensitivity analysis with scenario 1 having 4 cases, scenario 2 with 3 cases, scenario 3 with 2 cases and scenario 4 having only one case. In the first 4 cases in scenario 1, the weight of the most important decision maker (D_3) is alternated case by case with the weights of the remaining decision makers (DMs). In each instance of alternating the weight of the most important DM with another, the weights of all other DMs are held constant as shown in Table 35. For instance in scenario 1 case 1, W_{DD31}, W_{D2}, W_{D4}-W_{D7} means the interchange of weights of DMs 3 and 1 whiles that of DMs 2, 4, 5, 6 and 5 were held constant. In scenario 2 case 8, the expression W_{DD17}, W_{D2}-W_{D6} implies interchanging of weights between DMs 1 and 7 whiles maintaining the weights of DMs 2, 3, 4, 5 and 6 constant. This approach is replicated all throughout Table 35.

Table 35. Inputs for sensitivity analysis (Schema 2, Numerical example 1)

Case	**(Changes made in Wjt), J=DM$_1$, DM$_2$, …, DM$_n$, t=1,2,...,n**	
Scenario 1	Case 1	W_{DD31}, W_{D2}, W_{D4}-W_{D7}
	Case 2	W_{DD32}, W_{D1}, W_{D4}-W_{D7}
	Case 3	W_{DD34}, , W_{D1}-W_{D2}, W_{D5}-W_{D7}
	Case 4	W_{DD35}, W_{D1}-W_{D2}, W_{D4}, W_{D6}-W_{D7}
	Case 5	W_{DD36}, W_{D1}-W_{D2}, $W_{D4} - W_{D5}$, W_{D7}
	Case 6	W_{DD37}, W_{D1}-W_{D2}, W_{D4}-W_{D6}
Scenario 2	Case 7	W_{DD12}, W_{D3}-W_{D7}
	Case 8	W_{DD17}, W_{D2}-W_{D6}
Scenario 3	Case 9	W_{DD76}, W_{D1}-W_{D5}
Scenario 4	Case 10	W_{DD45}, W_{D1}-W_{D3}, W_{D6}-W_{D7}

In Table 36, the results of the sensitivity analysis based on the 10-cases in the 4-scenarios are shown along with the original ranking of the alternatives. The results indicate that the original ranking obtained remains unchanged in most part of the scenarios except in scenario 1 case 4 and 6 where alternative A1 outranks alternative A5 at fourth position as compared to the original ranking. It must be added that the results would always depend on the particular decision problem at hand.

Table 36. Results of sensitivity analysis (Schema 2, Numerical example 1)

		Scenario 1						Scenario 2		Scenario 3	Scenario 4
	Original Ranking	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
A1	**5**	5	5	5	4	5	4	5	5	5	5
A2	**3**	3	3	3	3	3	3	3	3	3	3
A3	**1**	1	1	1	1	1	1	1	1	1	1
A4	**2**	2	2	2	2	2	2	2	2	2	2
A5	**4**	4	4	4	5	4	5	4	4	4	4

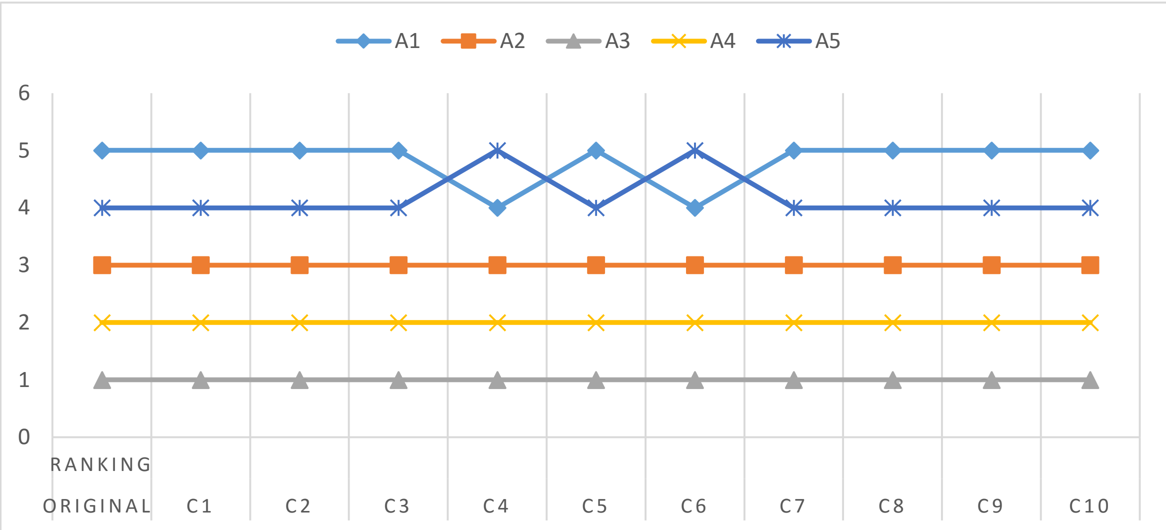


Fig. 23: Plot of sensitivity analysis (Schema 2, Numerical example 1)

The sensitivity analysis confirms that no matter the changes made to the weights of the decision makers, alternative A3 would remain the best candidate in the selection of a new manager.

Schema 2 - Numerical Example 2

The idea of swapping decision makers' weights as described in schema 2 is again investigated on numerical example 2. In numerical example 2, the order of importance of the decision makers were the following: $D_1 > D_2 > D_3 > D_4 > D_5$. Table 37 presents the inputs of the sensitivity analysis with scenario 1 having 4 cases, scenario 2 with 3 cases, scenario 3 with 2 cases and scenario 4 having only one case. In the first 4 cases in scenario 1, the weight of the most important decision maker (D_1) is alternated case by case with the weights of the remaining decision makers (DMs). In each instance of alternating the weight of the most important DM with another, the weights of all other DMs are held constant as shown and replicated through Table 37.

Table 37. Inputs for sensitivity analysis (Schema 2, Numerical example 2)

Case	**(Changes made in Wjt), J=DM$_1$, DM$_2$, …, DM$_n$, t=1,2,...,n**	
Scenario 1	Case 1	W_{DD12}, W_{D3}-W_{D5}
	Case 2	W_{DD13}, W_{D2}, W_{D4}-W_{D5}
	Case 3	W_{DD14}, W_{D2}-W_{D3}, W_{D5}
	Case 4	W_{DD15}, W_{D2}-W_{D4}
Scenario 2	Case 5	W_{DD23}, W_{D1}, W_{D4}-W_{D5}
	Case 6	W_{DD24}, W_{D1}, W_{D3}, W_{D5}
	Case 7	W_{DD25}, $W_{D1,}$ W_{D3},W_{D4}
Scenario 3	Case 8	W_{DD34}, W_{D1}-W_{D2}, W_{D5}
	Case 9	W_{DD35}, W_{D1}-W_{D2}, W_{D4}
Scenario 4	Case 10	W_{DD45}, W_{D1}-W_{D3}

The results of the sensitivity analysis based on schema 2 as applied on numerical example 2, is shown in Table 38 and figure 24. It can be seen that the ranking largely remains unchanged especially among the first 2 ideal alternatives. The changes that occur in the original ranking comes in cases 4, 5, 7 and 9 respectively.

Table 38. Results of sensitivity analysis (Schema 2, Numerical example 2)

		Scenario 1				Scenario 2			Scenario 3		Scenario 4
	Original Ranking	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
A1	**2**	2	2	2	2	2	2	2	3	2	2
A2	**3**	3	3	3	3	3	3	3	2	3	3
A3	**1**	1	1	1	1	1	1	1	1	1	1
A4	**4**	4	4	4	4	5	4	5	4	4	4
A5	**5**	5	5	5	5	4	5	4	5	5	5

The results after the sensitivity analysis still places A3 as the best alternative or the best distributing company in light of the proposed 2-tier hybrid decision model.

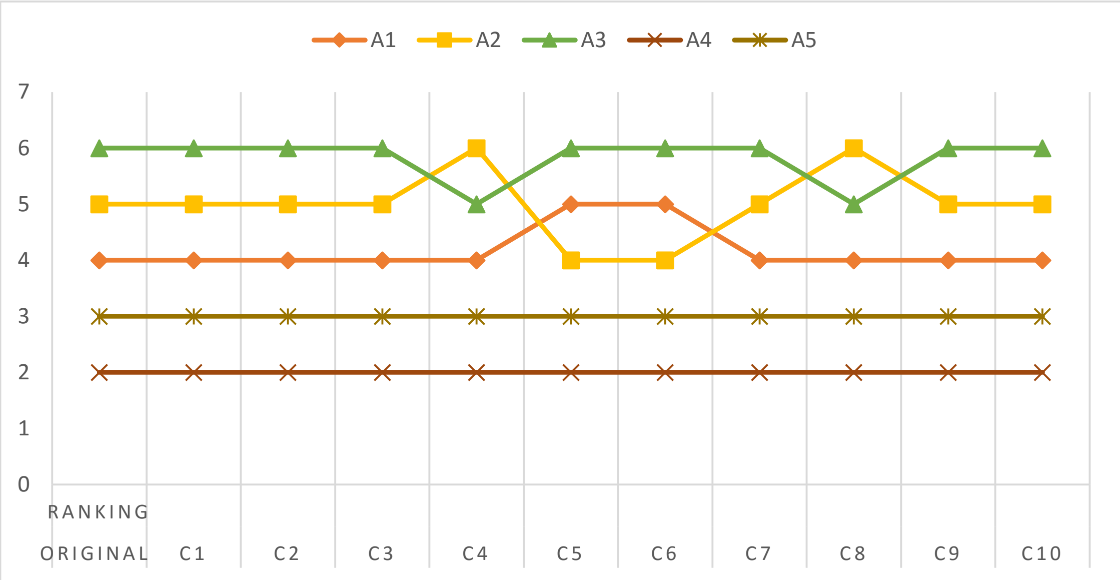


Fig. 24: Plot of sensitivity analysis (Schema 2, Numerical example 2)

Schema 2 - Numerical Example 3

In schema 2 numerical example 3, the inputs in Table 39 are compared to the original ranking results as achieved in Table 28. The order of importance of the decision makers were the following: $D_3 > D_2 = D_1 > D_4 > D_5$. In the first 4 cases in scenario 1, the weight of the most important decision maker (D_3) is alternated case by case with the weights of the remaining decision makers (DMs). This approach is repeated all throughout Table 39.

Table 39. Inputs for sensitivity analysis (Schema 2, Numerical example 3)

Case	**(Changes made in Wjt), J=DM$_1$, DM$_2$, …, DM$_n$, t=1,2,...,n**	
Scenario 1	Case 1	W_{DD31}, W_{D2}, W_{D4}-W_{D5}
	Case 2	W_{DD32}, W_{D1}, W_{D4}-W_{D5}
	Case 3	W_{DD34}, W_{D1}-W_{D2}, W_{D5}
	Case 4	W_{DD35}, W_{D1}-W_{D2}, W_{D4}
Scenario 2	Case 5	W_{DD21}, W_{D3}-W_{D5}
	Case 6	$W_{DD24}, W_{D1}, W_{D3}, W_{D5}$
	Case 7	$W_{DD25}, W_{D1,}\ W_{D3}, W_{D4}$
Scenario 3	Case 8	W_{DD41}, W_{D2}-W_{D3}, W_{D5}
	Case 9	W_{DD45}, W_{D1}-W_{D3}
Scenario 4	Case 10	W_{DD51}, W_{D2}-W_{D4}

The results in Table 40 and figure 25 show some changes to the original ranking as a result of the changes to decision makers' weights. These changes however are seen among alternatives 4, 5 and 6. This means that if the goal of the decision making was to select only one ideal alternative, then these changes recorded, would not significantly affect the original ranking. Alternative 6 is still affirmed the best alternative even after the sensitivity analysis.

Table 40. Results of sensitivity analysis (Schema 2, Numerical example 3)

		Scenario 1				Scenario 2			Scenario 3		Scenario 4
	Original Ranking	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
A1	**4**	4	4	4	4	5	5	4	4	4	4
A2	**5**	5	5	5	6	4	4	5	6	5	5
A3	**6**	6	6	6	5	6	6	6	5	6	6
A4	**2**	2	2	2	2	2	2	2	2	2	2
A5	**3**	3	3	3	3	3	3	3	3	3	3
A6	**1**	1	1	1	1	1	1	1	1	1	1

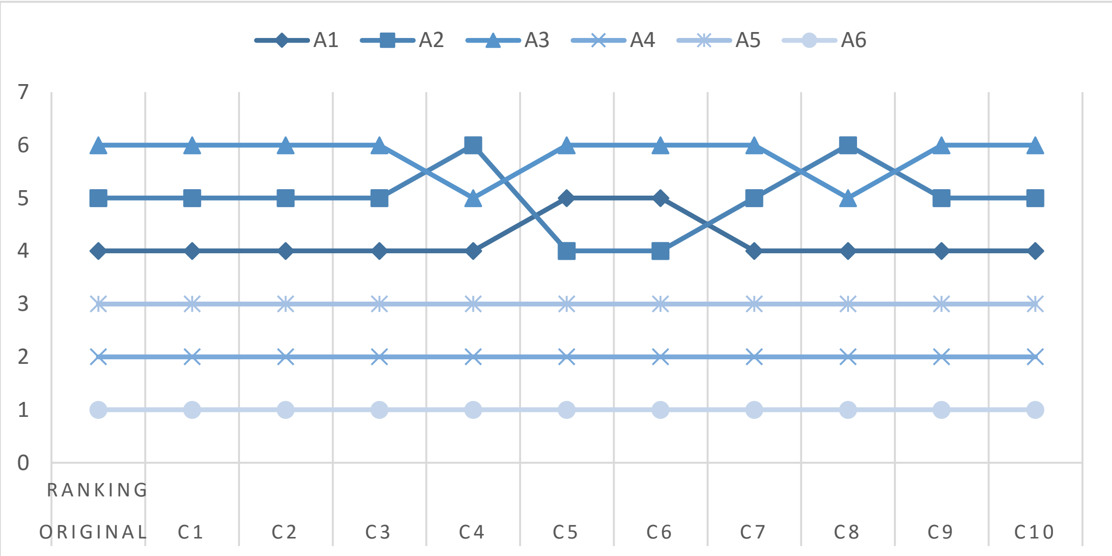


Fig. 25: Plot of sensitivity analysis (Schema 2, Numerical example 3)

10.3 Schema 3: Swapping both criteria and decision makers' weights concurrently

In schema 3, the option of changing concurrently the weights of criteria as well as decision makers is investigated. It must be noted that this is a special scenario in multi-criteria decision making where both criteria and decision makers' are assigned weights. To explore this situation, the inputs used in the first two schemas for each numerical examples are merged into one novel model where both the weights of criteria and the decision makers are swapped case by case with their respective sets. In this schema, the model is tested on only the third numerical example. There are 10 input cases under 4 scenarios tested on each numerical example. In each instance of swapping the weights of the most important criterion and decision makers with their respective members, weights of all the other criteria in the particular sets are held constant. For instance the expression, W_{DD31},W_{D2},W_{D4}-W_{D5} ∧ WCC_{21},Wc3-Wc5 would mean the alternation of the weights of decision makers 3 and 1; criteria 2 and 1 whiles the weights of the rest of the elements in each set of decision makers and criteria are held constant. The weights swapping is conducted for all the rest of the criteria and decision makers in similar manner.

Schema 3 - Numerical Example 3

In numerical example 3, the decision problem was to select the best recruitment agency as an HR partner to a manufacturing company using the proposed conjoint analysis-intuitionistic fuzzy TOPSIS method. The decision making process realized the following order of importance $C_2 > C_4 > C_1 > C_5 > C_3$, $D_3 > D_2 = D_1 > D_4 > D_5$ respectively for the criteria and the decision makers. In Table 41, the inputs of the sensitivity analysis for schema 1 as tested on numerical example 3 are presented. In the first 4 cases in scenario 1, the weight of the most important criteria (C_2) and decision maker (D_3) are concurrently alternated case by case with the weights of their respective elements in the set. Furthermore, the rest of the elements in the set of either criteria or decision makers are held constant whiles the swapping is been carried out.

Table 41. Inputs for sensitivity analysis (Schema 3, Numerical example 3)

Case	**(Changes made in Wjt),** ***i*=DM$_1$, DM$_2$, …, DM$_n$, *J*= C1, C2, …, Cn, t=1,2,...,n**	
Scenario 1	Case 1	WCC_{21}, Wc3-Wc5 ∧ W_{DD31}, W_{D2}, W_{D4}-W_{D5}
	Case 2	WCC_{23}, Wc1, Wc4-Wc5 ∧ W_{DD32}, W_{D1}, W_{D4}-W_{D5}
	Case 3	WCC_{24}, Wc1, Wc3,Wc5 ∧ W_{DD34}, W_{D1}-W_{D2}, W_{D5}
	Case 4	WCC_{25}, Wc1, Wc3-Wc4 ∧ W_{DD35}, W_{D1}-W_{D2}, W_{D4}
Scenario 2	Case 5	W_{DD21}, W_{D3}-W_{D5} ∧ WCC_{41}, Wc2-Wc3, Wc5
	Case 6	WCC_{45}, Wc1- Wc3 ∧ W_{DD24}, W_{D1}, W_{D3}, W_{D5}
	Case 7	WCC_{43}, Wc1- Wc2, Wc5 ∧ W_{DD25}, W_{D1}, W_{D3},W_{D4}
Scenario 3	Case 8	WCC_{15}, Wc2-Wc4 ∧ W_{DD41}, W_{D2}-W_{D3}, W_{D5}
	Case 9	WCC_{13}, Wc2, Wc4-Wc5 ∧ W_{DD45}, W_{D1}-W_{D3}
Scenario 4	Case 10	WCC_{35}, Wc1-Wc2, Wc4 ∧ W_{DD51}, W_{D2}-W_{D4}

In Table 42, the results of the sensitivity analysis based on the 10-cases in the 4-scenarios are shown along with the original ranking of the alternatives. The results indicate that the original ranking obtained remains unchanged in most part of the scenarios except in scenario 1 case 4 and 6 where alternative A1 outranks alternative A5 at fourth position as compared to the original ranking. It must be added that the results would always depend on the particular decision problem at hand.

Table 42. Results of sensitivity analysis (Schema 3, Numerical example 3)

		Scenario 1				Scenario 2			Scenario 3		Scenario 4
	Original Ranking	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	**4**	4	4	4	4	4	5	4	4	4	5
A2	**5**	5	5	5	5	5	4	4	5	5	4
A3	**6**	6	6	6	6	6	6	5	6	6	6
A4	**2**	2	3	2	3	2	2	3	2	3	2
A5	**3**	3	2	3	2	3	3	2	3	2	3
A6	**1**	1	1	1	1	1	1	1	1	1	1

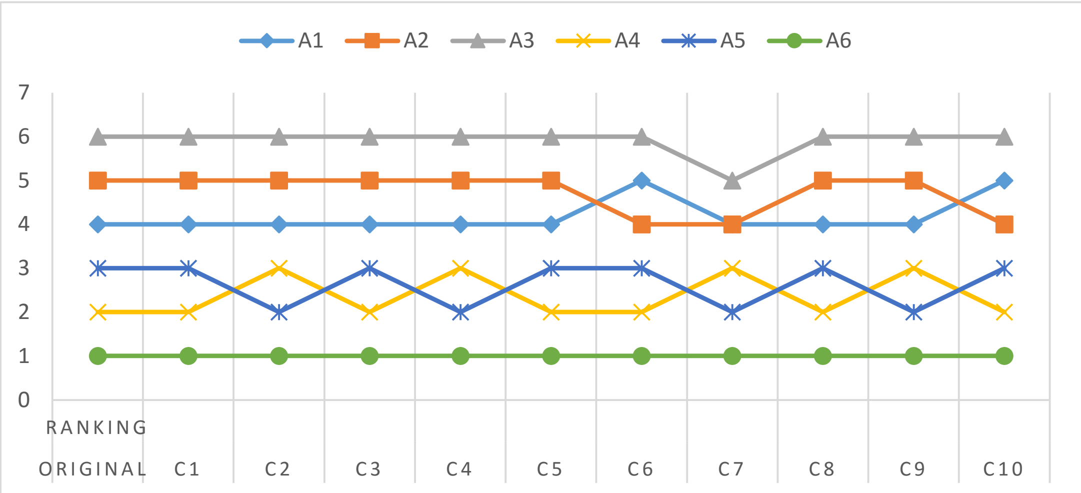


Fig. 26: Plot of sensitivity analysis (Schema 3, Numerical example 3)

11 CONCLUSION AND DISCUSSION

Research in Multi-criteria decision making (MCDM) has come of age with growing number of methods and approaches used for optimal decision making. The use of fuzzy sets which extend the scope of solutions in MCDM, especially for uncertainty modelling, helps to bridge the array of tools between computer science and operations research. Such multidisciplinary approach to MCDM has helped to usher in wide array of tools with some outside the scope of MCDM. Some of the multidisciplinary approaches in MCDM combine several methods at various stages in the multi-criteria decision steps. This integration of methods are popularly known as 'hybrid' methods. In recent times the use of such hybrid methods has increased significantly. However, the choice and appropriateness of hybrid methods over single method solutions especially in MCDM applications, are most often not fully explained.

This dissertation was organized into two parts. The first investigated the trend of use of fuzzy hybrid MCDM methods as well as the appropriateness of such methods over single MCDM method solutions. The use of hybrid and single or one-method solutions were compared. The results showed that whiles literature is abound lately with many hybrid MCDM solutions, more often, a single method also achieves the same or similar results especially regarding the final ranking of alternatives. However the results also demonstrated that the ranking order of alternatives in a single MCDM method solutions compared with a hybrid method, tends to be different when the two methods in the 'hybrid' come from different categories or types of MCDM methods. For instance when the hybrid is composed of one method from a full aggregation background (American School) and the other from an outranking method (French School), the results (ranking order) changes when one of the methods in the hybrid is used. Such irresolution or inconclusiveness in the results demonstrated that hybrid methods in MCDM must be treated with care and used in special decision problems. Based on results of the investigative part of the research, a second part proposed a hybrid MCDM method and demonstrated its use in special decision problems.

The proposed hybrid MCDM framework introduced in the second part of the dissertation mirrors ideal scenarios of real-world decision problems. With 3 numerical examples, the proposed 2-tier hybrid decision framework demonstrates how two decision points; one based on preferences from a large decision making group and the other from a relatively small group can be merged to achieve an optimal decision making. Conjoint analysis and intuitionistic fuzzy TOPSIS methods are the tools in the proposed hybrid method. The proposed method uses these two existing methods; one from marketing research and the other, a fuzzy extension of TOPSIS to demonstrate the applicability of such hybrid method. The conjoint analysis was used to model preference data of a relatively lager decision making group where trade-offs are necessary to be made in decisions. The preference data are converted into criteria weights and subsequently merged with decisions from a relatively small group (experts). The intuitionistic fuzzy TOPSIS method aided in the ranking and selection of alternatives. The intuitionistic fuzzy TOPSIS method is specially selected because of its ability to readily reveal or impugn meaning into decision makers' ratings. The inherent meaning behind a decision maker's rating is known through the use of the intuitionistic fuzzy sets. TOPSIS helped to separate ideal solutions (alternatives) from anti-ideal solutions.

The proposed method was tested on 3 numerical examples to demonstrate the strength and the special use of such method. In the first numerical example, the proposed conjoint analysis – intuitionistic fuzzy TOPSIS framework is demonstrated in a decision problem involving incorporating shareholder preferences into management decisions. This decision problem is premised on the growing power struggle between shareholders of companies and their management boards. The proposed decision making framework demonstrates how shareholder preferences can be incorporated into management decisions regarding the selection of a new manager in a microfinance organization. Since shareholders tend to be many in number, their views and preferences on characteristics of 'their' ideal candidate are modelled into criteria weights to guide management in their final selection of the best candidate. Further, board of management uses the intuitionistic fuzzy TOPSIS method to rank and select the ideal candidate for the vacant managerial position.

In the numerical example 2, the decision solution was unique in how customer preferences could be incorporated into management decision about selecting an ideal third party distributing company based on their previous performances. Similar to problem 1, conjoint analysis – intuitionistic fuzzy TOPSIS was ideal for such problem. In numerical example 3, similar approach of testing the proposed approach was carried out with a decision problem involving merging managers' preferences and trade-offs in recruitment process outsourcing vendor selection decisions. The conjoint analysis – intuitionistic fuzzy TOPSIS framework again proved reliable and adequate for such special decision problems of incorporating two decision points into a composite one. It must be noted that the numerical examples provide a guide to how the proposed hybrid method can be extended into other real-life decision problems.

The last stage in the decision framework in this dissertation focused on providing novel sensitivity analysis models that can be used to test the reliability of the final decisions and by extension the congruent effect produced when certain changes are made to the input parameters. Three models conveniently named as 'schemas' were developed for the sensitivity analysis. The sensitivity analysis model can be extended into other real-life decisions.

The extensive literature review on hybrid fuzzy MCDM methods revealed in two ways how the concept of 'hybrid' MCDM are typically approached. Hybrid MCDM normally (1) describes the nature of the problem and (2) the nature of the solution (methods). This dissertation focused on the aspect relating to the latter concerning methods and solutions used. In future works, the researcher hopes to investigate into hybrid MCDM methods regarding the nature of the underlying problem.

The results obtained in both the investigative part and in the proposed hybrid method, give reason to believe that hybrid MCDM methods should only be used in special decision problems. Additionally, it should be used only when the use of a single MCDM method would not be adequate. As demonstrated in this dissertation, the proposed hybrid method was used in some of these complex and challenging decision problems.

One **remarkable strength** of the dissertation is the introduction of conjoint analysis method which is typically outside MCDM methods and demonstrating its similarities to most MCDM methods. Though the two methods used in the proposed hybrid solution are existing methods from different fields of disciplines, harnessing their strengths in this unique 2-tier hybrid decision making model has been novel. The novel integration of conjoint analysis and intuitionistic fuzzy TOPSIS could spur further research into discovering other methods outside the scope of MCDM methods that could be paired seamlessly with MCDM methods to solve real-life problems. Furthermore, the schemas designed for the sensitivity analysis also provide a means to understanding variety of ways of modelling sensitivity analysis in MCDM.

Overall, the dissertation was successful both in the investigative and the design aspects. The investigative part offered the chance to understand the use of hybrid methods especially in relation to single-method solutions. The proposed hybrid conjoint analysis-intuitionistic fuzzy TOPSIS method compared favourably with other hybrid fuzzy MCDM methods. In particular, the conjoint analysis method faired better compared to AHP regarding criteria weight setting from the relatively large decision group. It can therefore be concluded that conjoint analysis is a better choice than AHP when the number of preference inputs are large. It must however be added that, quality of results achieved in any MCDM exercise, depends largely on the complexity of the evaluation task and the knowledge of the respondent/decision maker regarding preference measurement. Again, in terms of time complexity, AHP is slower compared to conjoint analysis because of its many pairwise comparisons.

The aims of the dissertation were achieved:

- *To investigate the use of hybridized or integrated MCDM methods against one-method solutions. To determine:*
  - *when a hybrid method solution is useful to a selection problem.*
  - *which MCDM method is ideal for setting criteria weights in a hybridized method.*
  - *the appropriate use of subjective and objective weights in a hybridized method for a selection problem.*

The investigative part of the dissertation offered the researcher the chance to compare hybrid MCDM methods with single MCDM methods on same decision problems. Through the investigation, it can be concluded that hybrid MCDM methods should be deployed only in very special cases where a one-method solution would not be helpful. It was also observed that the AHP method has superior qualities for setting criteria weights but when decision inputs are relatively large and trade-offs need to be modelled, conjoint analysis performs better with clearer results for interpretation.

- *Design a new hybridized MCDM method that embodies user (consumers, shareholder) preferences and experts in an ideal decision making situation.*
    - *Conjoint Analysis – Intuitionistic Fuzzy TOPSIS Method*
    - *Conjoint Analysis for setting criteria weights*
    - *Intuitionistic Fuzzy TOPSIS for ranking competing alternatives*
    - *Test the proposed hybrid MCDM model with real-life examples*

A new hybrid MCDM method was designed from two existing methods which was adequately tested on numerical examples. The proposed hybrid method compares favourably with other hybrid MCDM methods. The strength of the proposed method is its ability to seamlessly merge two decision points into a composite decision.

- *Test the proposed hybrid MCDM model with real-life examples.*

The proposed hybrid decision model was tested on numerical examples that mirror real-life situations. Three examples were demonstrated on how decisions of preferences from a large decision making group are merged with decisions from a relatively small decision making group. The reliability of final decisions in each numerical example was tested using novel sensitivity analysis models.

The author would like to add that most of the results in the dissertation have been published in several conferences and impacted journals such as *Applied Artificial Intelligence, Kybernetes, Quality & Quantity, Journal of Multi-Criteria Decision Analysis, International Journal of Fuzzy Systems* as seen in the list of publications.

REFERENCES

[1] Gwo-Hshiung, T. (2010). Multiple attribute decision making: methods and applications. Multiple Attribute Decision Making: Methods and Applications. ISBN: 1439861579

[2] Triantaphyllou, E. (2000). Multi-criteria decision making methods. In Multi-criteria Decision Making Methods: A Comparative Study (pp. 5-21). Springer US. ISBN-13: 978-0792366072.

[3] Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), 338-353.

[4] Afful-Dadzie, E., Nabareseh, S., Afful-Dadzie, A., & Oplatková, Z. K. (2014). A fuzzy TOPSIS framework for selecting fragile states for support facility. Quality & Quantity, 49 (5), 1835-1855. ISSN 0033-5177.

[5] Kahraman, C. (2008). Fuzzy multi-criteria decision making: theory and applications with recent developments (Vol. 16). Springer Science & Business Media. ISBN 978-0-387-76813-7

[6] Szmidt, E. (2014). Distances and similarities in intuitionistic fuzzy sets. Springer. ISBN 978-3-319-01640-5.

[7] Zadeh, L. A. (1983). A computational approach to fuzzy quantifiers in natural languages. Computers & Mathematics with applications, 9(1), 149-184. ISSN: 0898-1221.

[8] Pawlak, Z. (1982). Rough sets. International Journal of Computer & Information Sciences, 11(5), 341-356. ISSN:1525-9293.

[9] Dempster, A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. The annals of mathematical statistics, 325-339.

[10] Dempster, A. P. (1967). Upper and lower probability inferences based on a sample from a finite univariate population. Biometrika, 54(3-4), 515-528. ISBN-13: 978-0198509936.

[11] Shafer, G. (1976). A mathematical theory of evidence (Vol. 1). Princeton: Princeton university press. ISBN: 9780691100425.

[12] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy sets and Systems, 20(1), 87-96. ISSN: 0165-0114.

[13] Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. Fuzzy sets and systems, 33(1), 37-45. ISSN: 0165-0114.

[14] Atanassov, K. T. (1994). New operations defined over the intuitionistic fuzzy sets. Fuzzy sets and Systems, 61(2), 137-142. ISSN: 0165-0114.

[15] Keeney, R. L., & Raiffa, H. (1993). Decisions with multiple objectives: preferences and value trade-offs. Cambridge university press. ISBN 0-521-44185-4.

[16] Kleindorfer, P.R., H.C. Kunreuther, and P.J.H. Schoemaker. (1993). Decision sciences: An integrative perspective. Cambridge: Cambridge University Press. ISBN 0-521-32867-5.

[17] Czogała, E., & Łęski, J. (2000). Classical sets and fuzzy sets Basic definitions and terminology. In Fuzzy and Neuro-Fuzzy Intelligent Systems (pp. 1-26). Physica-Verlag HD. ISBN: 978-3-662-00389-3.

[18] Carlsson, C., & Fullér, R. (2005). On additions of interactive fuzzy numbers. Acta Polytechnica Hungarica, 2, 59-73. ISSN: 17858860.
[19] Kaufmann, A., & Gupta, M. M. (1991). Introduction to fuzzy arithmetic: theory and applications. Arden Shakespeare. ISBN-10: 0442230079.
[20] Boran, F. E., Genç, S., Kurt, M., & Akay, D. (2009). A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Systems with Applications, 36(8), 11363-11368. ISSN: 09574174.
[21] Carlsson, C., & Fullér, R. (1996). Fuzzy multiple criteria decision making: Recent developments. Fuzzy sets and systems, 78(2), 139-153. ISSN: 0165-0114.
[22] Chu, T. C., & Lin, Y. (2009). An extension to fuzzy MCDM. Computers & Mathematics with Applications, 57(3), 445-454. ISSN: 0898-1221.
[23] Ribeiro, R. A. (1996). Fuzzy multiple attribute decision making: a review and new preference elicitation techniques. Fuzzy sets and systems, 78(2), 155-181. ISSN: 0165-0114.
[24] Büyüközkan, G., & Çifçi, G. (2012). A novel hybrid MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. Expert Systems with Applications, 39(3), 3000-3011. ISSN: 09574174.
[25] Zhang, S. F., & Liu, S. Y. (2011). A GRA-based intuitionistic fuzzy multi-criteria group decision making method for personnel selection. Expert Systems with Applications, 38(9), 11401-11405. ISSN: 09574174.
[26] Wu, J., Chen, F., Nie, C., & Zhang, Q. (2013). Intuitionistic fuzzy-valued Choquet integral and its application in multi-criteria decision making. Information Sciences, 222, 509-527. ISSN: 00200255.
[27] Chai, J., Liu, J. N., & Xu, Z. (2012). A new rule-based SIR approach to supplier selection under intuitionistic fuzzy environments. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 20(03), 451-471. ISSN: 0218-4885.
[28] Chen, S. M., Yang, M. W., Yang, S. W., Sheu, T. W., & Liau, C. J. (2012). Multi-criteria fuzzy decision making based on interval-valued intuitionistic fuzzy sets. Expert Systems with Applications, 39(15), 12085-12091. ISSN: 09574174.
[29] Krohling, R. A., Pacheco, A. G., & Siviero, A. L. (2013). IF-TODIM: An intuitionistic fuzzy TODIM to multi-criteria decision making. Knowledge-Based Systems, 53, 142-146. ISSN: 09507051.
[30] Hashemi, H., Bazargan, J., & Mousavi, S. M. (2013). A compromise ratio method with an application to water resources management: An intuitionistic fuzzy set. Water resources management, 27(7), 2029-2051. ISSN: 09204741.

[31] Meng, F., Tan, C., & Zhang, Q. (2014). Some interval-valued intuitionistic uncertain linguistic hybrid Shapley operators. Systems Engineering and Electronics, Journal of, 25(3), 452-463. ISSN: 10044132.
[32] Zhang, X. M., Xu, Z. S., & Yu, X. H. (2011). Shapley value and Choquet integral-based operators for aggregating correlated intuitionistic fuzzy information. Information: An International Interdisciplinary Journal, 14(6), 1847-1858.
[33] Chen, T. Y., & Li, C. H. (2010). Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis. Information Sciences, 180(21), 4207-4222. ISSN: 00200255
[34] Daneshvar Rouyendegh, B. (2011). The DEA and intuitionistic fuzzy TOPSIS approach to departments' performances: a pilot study. Journal of Applied Mathematics, 2011. ISSN: 16870042
[35] Li, D. F. (2014). Decision and game theory in management with intuitionistic fuzzy sets (Vol. 308, pp. 1-441). Springer. ISBN 978-3-642-40712-3.
[36] Xu, Z. (2007). Intuitionistic fuzzy aggregation operators. Fuzzy Systems, IEEE Transactions on, 15(6), 1179-1187. ISSN: 10636706.
[37] Chen, S. M., & Tan, J. M. (1994). Handling multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy sets and Systems, 67(2), 163-172. ISSN: 0165-0114.
[38] Zeng, S., & Su, W. (2011). Intuitionistic fuzzy ordered weighted distance operator. Knowledge-Based Systems, 24(8), 1224-1232. ISSN: 09507051.
[39] Yang, W., & Chen, Z. (2012). The quasi-arithmetic intuitionistic fuzzy OWA operators. Knowledge-Based Systems, 27, 219-233. ISSN: 09507051.
[40] Yager, R. R. (2001). The power average operator. Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 31(6), 724-731. ISBN 0780309111.
[41] Merigó, J. M. (2012). Probabilities in the OWA operator. Expert Systems with Applications, 39(13), 11456-11467. ISSN: 09574174.
[42] Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. Systems, Man and Cybernetics, IEEE Transactions on, 18(1), 183-190. ISBN 0780309111.
[43] Yager, R. R. (2004). Generalized OWA aggregation operators. Fuzzy Optimization and Decision Making, 3(1), 93-107. ISSN: 15732908.
[44] Yager, R. R. (2003). Induced aggregation operators. Fuzzy Sets and Systems, 137(1), 59-69. ISSN: 0165-0114.
[45] Saaty, T. L. (1990). How to make a decision: the analytic hierarchy process. European journal of operational research, 48(1), 9-26. ISSN: 03772217.
[46] Saaty, T. L. (2001). Analytic network process. In Encyclopedia of Operations Research and Management Science (pp. 28-35). Springer US.

[47] Hwang, C., & Yoon, K. (1981). Multiple attribute decision making methods and application, Springer, New York. ISBN 978-3-642-48318-9.
[48] Opricovic, S., & Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. European Journal of Operational Research, 178(2), 514-529. ISSN: 03772217.
[49] Chou, S. Y., Chang, Y. H., & Shen, C. Y. (2008). A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. European Journal of Operational Research, 189(1), 132-145. ISSN: 03772217.
[50] Roy, B. (1978). ELECTRE III: Un algorithme de classements fondé sur une représentation floue des préférences en présence de criteres multiples. Cahiers du CERO, 20(1), 3-24.
[51] Brans, J. P., Vincke, P., & Mareschal, B. (1986). How to select and how to rank projects: The PROMETHEE method. European journal of operational research, 24(2), 228-238. ISSN: 03772217.
[52] Chang, B., Chang, C. W., & Wu, C. H. (2011). Fuzzy DEMATEL method for developing supplier selection criteria. Expert Systems with Applications, 38(3), 1850-1858. ISSN: 09574174.
[53] Roy, B. (1981). The optimisation problem formulation: Criticism and overstepping. Journal of the Operational Research Society, 32(6), 427–436. ISSN: 14769360.
[54] Roy, B., & Słowiński, R. (2013). Questions guiding the choice of a multicriteria decision aiding method. EURO Journal on Decision Processes, 1(1-2), 69-97. ISSN: 2193-9438
[55] Hanne, T. (2012). Intelligent strategies for meta multiple criteria decision making (Vol. 33). Springer Science & Business Media. ISBN 978-1-4615-1595-1.
[56] Ishizaka, A., & Nemery, P. (2013). Multi-criteria decision analysis: methods and software. John Wiley & Sons. ISBN: 1119974070
[57] Bell, D.E., H. Raiffa, and A. Tversky. (1988). Decision making: Descriptive, normative, and prescriptive interactions. Cambridge: Cambridge University Press. isbn: 9780521368513.
[58] Pawlak, Z., 1982. Rough sets. International Journal of Computer & Information Sciences, 11(5), 341-356. ISSN:1525-9293.
[59] Grattan-Guinness, I. (1976). Fuzzy Membership Mapped onto Intervals and Many-Valued Quantities. Mathematical Logic Quarterly, 22(1), 149-160. ISSN: 15213870.
[60] Torra, V. (2010). Hesitant fuzzy sets. International Journal of Intelligent Systems, 25(6), 529-539. ISSN: 1098111X.
[61] Molodtsov, D. (1999). Soft set theory—first results. Computers & Mathematics with Applications, 37(4), 19-31. ISSN: 0898-1221.

[62] Zadeh, L. A. (1974). The concept of a linguistic variable and its application to approximate reasoning (pp. 1-10). Springer US. ISBN: 978-1-4684-2108-8
[63] Zadeh, L. A. (1975). Fuzzy logic and approximate reasoning. Synthese, 30(3-4), 407-428. ISSN: 0039-7857
[64] Zeleny, M. (2008). The KM-MCDM interface in decision design: tradeoffs-free conflict dissolution. International Journal of Applied Decision Sciences, 1(1), 3-23. ISSN: 17558085.
[65] Sawaragi, Y., Nakayama, H., & Tanino, T. (1985). Theory of multiobjective optimization, Acad.
[66] Hwang, C. L., & Masud, A. S. M. (1979). Multiple objective decision making—methods and applications. ISBN: 3540091114
[67] Zeleny, M. (1982). Multi criteria decision making. TIMS Studies in Manage, 31-57. ISBN 10: 0720407648.
[68] Vincke, P. (1992). Multicriteria decision-aid. John Wiley & Sons. ISBN: 978-0-471-93184-3
[69] Steuer, R. E. (1986). Multiple criteria optimization: theory, computation, and applications. Wiley. ISBN-10: 047188846X.
[70] Costa, E., Bana, C. A., & Vansnick, J. C. (1994). MACBETH—An interactive path towards the construction of cardinal value functions. International transactions in operational Research, 1(4), 489-500. ISSN: 09696016.
[71] Hansen, P., & Ombler, F. (2008). A new method for scoring additive multi-attribute value models using pairwise rankings of alternatives. Journal of Multi-Criteria Decision Analysis, 15(3-4), 87-107. ISSN: 1099-1360.
[72] Xu, X. (2001). The SIR method: A superiority and inferiority ranking method for multiple criteria decision making. European Journal of Operational Research, 131(3), 587-602. ISSN: 03772217.
[72] Xu, L., & Yang, J. B. (2001). Introduction to multi-criteria decision making and the evidential reasoning approach. Manchester School of Managment, University of Manchester Institute of Science and Technology.
[73] Dyer, J. S., Fishburn, P. C., Steuer, R. E., Wallenius, J., & Zionts, S. (1992). Multiple criteria decision making, multiattribute utility theory: the next ten years. Management science, 38(5), 645-654. ISSN: 15265501.
[74] Geoffrion, A. M. (1968). Proper efficiency and the theory of vector maximization. Journal of Mathematical Analysis and Applications, 22(3), 618-630. ISSN: 10960813.
[75] Evans, J. P., & Steuer, R. E. (1973). A revised simplex method for linear multiple objective programs. Mathematical Programming, 5(1), 54-72. ISSN: 14364646.

[76] Yu, P. L., & Zeleny, M. (1975). The set of all non-dominated solutions in linear cases and a multicriteria simplex method. Journal of Mathematical Analysis and Applications, 49(2), 430-468. ISSN: 10960813.
[77] Gal, T. (1977). A general method for determining the set of all efficient solutions to a linear vector maximum problem. European journal of operational research, 1(5), 307-322. ISSN: 03772217.
[78] Isermann, H. (1977). The enumeration of the set of all efficient solutions for a linear multiple objective program. Operational Research Quarterly, 711-725.
[79] Bitran, G. R. (1979). Theory and algorithms for linear multiple objective programs with zero–one variables. Mathematical Programming, 17(1), 362-390. ISSN: 14364646.
[80] Nakayama, H., & Sawaragi, Y. (1984). Satisficing trade-off method for multiobjective programming. In Interactive decision analysis (pp. 113-122). Springer Berlin Heidelberg. ISBN: 978-3-540-13354-4.
[81] Benayoun, R., De Montgolfier, J., Tergny, J., & Laritchev, O. (1971). Linear programming with multiple objective functions: Step method (STEM). Mathematical programming, 1(1), 366-375.
[82] Simon, H. A. (1950). Administrative behaviour. Australian Journal of Public Administration, 9(1), 241-245. ISSN: 03136647.
[83] Lakhmi, J., & Lim, C. P. (2010). Handbook on Decision Making: Techniques and Applications. Intelligent Systems Reference Library, 4, 548. ISBN 978-3-642-13639-9.
[84] Hajkowicz, S. A., McDonald, G. T., & Smith, P. N. (2000). An evaluation of multiple objective decision support weighting techniques in natural resource management. Journal of Environmental Planning and Management, 43(4), 505-518. ISSN: 09640568.
[85] Bouyssou, D. (1993). Décision multicritère ou aide multicritère. Newsletter of the European Working Group—Multicriteria Aid for Decisions, 1-2.
[86] Guitouni, A., Martel, J. M., & Bélanger, M. (1999). A multiple criteria aggregation procedure for the evaluation of courses of action in the context of the Canadian Airspace Protection. DREV TR, 1999-215.
[87] Kao, C., & Liu, S. T. (2000). Fuzzy efficiency measures in data envelopment analysis. Fuzzy sets and systems, 113(3), 427-437.
[88] Lertworasirikul, S., Fang, S. C., Joines, J. A., & Nuttle, H. L. (2003). Fuzzy data envelopment analysis (DEA): a possibility approach. Fuzzy sets and systems, 139(2), 379-394. ISSN: 0165-0114.
[89] Yoon, K. P., & Hwang, C. L. (1995). Multiple attribute decision making: an introduction (Vol. 104). Sage publications. ISBN: 0803954867.
[90] Sivanandam, S. N., Sumathi, S., & Deepa, S. N. (2007). Introduction to fuzzy logic using MATLAB (Vol. 1). Berlin: Springer. ISBN-13: 978-3642071447.

[91] Zadeh, L. A., Klir, G. J., & Yuan, B. (1996). Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers (Vol. 6). World Scientific. ISBN 9810224214.
[92] Klir, G., & Yuan, B. (1995). Fuzzy sets and fuzzy logic (Vol. 4). New Jersey: Prentice Hall. ISBN-13: 978-0131011717.
[93] Singh, H., Gupta, M. M., Meitzler, T., Hou, Z. G., Garg, K. K., Solo, A. M., & Zadeh, L. A. (2013). Real-life applications of fuzzy logic. Advances in Fuzzy Systems. ISSN: 1687711X.
[94] Yu, D., & Shi, S. (2015). Researching the development of Atanassov intuitionistic fuzzy set: Using a citation network analysis. Applied Soft Computing, 32, 189-198. ISSN: 15684946.
[95] Chen, T. Y. (2015). IVIF-PROMETHEE outranking methods for multiple criteria decision analysis based on interval-valued intuitionistic fuzzy sets. Fuzzy Optimization and Decision Making, 14(2), 173-198. ISSN: 15732908.
[96] Pourahmad, A., Hosseini, A., Banaitis, A., Nasiri, H., Banaitienė, N., & Tzeng, G. H. (2015). Combination of fuzzy-AHP and DEMATEL-ANP with GIS in a new hybrid MCDM model used for the selection of the best space for leisure in a blighted urban site. Technological and Economic Development of Economy, 21(5), 773-796. ISSN: 13928619.
[97] Su, C. H., Tzeng, G. H., & Tseng, H. L. (2012, November). Improving cloud computing service in fuzzy environment—combining fuzzy DANP and fuzzy VIKOR with a new hybrid FMCDM model. In Fuzzy Theory and it's Applications (iFUZZY), 2012 International Conference on (pp. 30-35). IEEE.
[98] Büyüközkan, G., & Çifçi, G. (2012). A novel hybrid MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. Expert Systems with Applications, 39(3), 3000-3011. ISSN: 09574174.
[99] Tzeng, G. H., Chiang, C. H., & Li, C. W. (2007). Evaluating intertwined effects in e-learning programs: A novel hybrid MCDM model based on factor analysis and DEMATEL. Expert systems with Applications, 32(4), 1028-1044. ISSN: 09574174.
[100] Hsu, C. H., Wang, F. K., & Tzeng, G. H. (2012). The best vendor selection for conducting the recycled material based on a hybrid MCDM model combining DANP with VIKOR. Resources, Conservation and Recycling, 66, 95-111. ISSN: 09213449.
[101] Lee, A. H., Chen, W. C., & Chang, C. J. (2008). A fuzzy AHP and BSC approach for evaluating performance of IT department in the manufacturing industry in Taiwan. Expert systems with applications, 34(1), 96-107. ISSN: 09574174.
[102] Li, M., Jin, L., & Wang, J. (2014). A new MCDM method combining QFD with TOPSIS for knowledge management system selection from the

user's perspective in intuitionistic fuzzy environment. Applied soft computing, 21, 28-37. ISSN: 15684946.

[103] Azimi, R., Yazdani-Chamzini, A., Fouladgar, M. M., Zavadskas, E. K., & Basiri, M. H. (2011). Ranking the strategies of mining sector through ANP and TOPSIS in a SWOT framework. Journal of Business Economics and Management, 12(4), 670-689. ISSN: 20294433.

[104] Chen, M. K., & Wang, S. C. (2010). The use of a hybrid fuzzy-Delphi-AHP approach to develop global business intelligence for information service firms. Expert Systems with Applications, 37(11), 7394-7407. ISSN: 09574174.

[105] Dalalah, D., Hayajneh, M., & Batieha, F. (2011). A fuzzy multi-criteria decision making model for supplier selection. Expert systems with applications, 38(7), 8384-8391. ISSN: 09574174.

[106] Paksoy, T., Pehlivan, N. Y., & Kahraman, C. (2012). Organizational strategy development in distribution channel management using fuzzy AHP and hierarchical fuzzy TOPSIS. Expert Systems with Applications, 39(3), 2822-2841. ISSN: 09574174.

[107] Patil, S. K., & Kant, R. (2014). A fuzzy AHP-TOPSIS framework for ranking the solutions of Knowledge Management adoption in Supply Chain to overcome its barriers. Expert Systems with Applications, 41(2), 679-693. ISSN: 09574174.

[108] Taha, Z., & Rostam, S. (2012). A hybrid fuzzy AHP-PROMETHEE decision support system for machine tool selection in flexible manufacturing cell. Journal of Intelligent Manufacturing, 23(6), 2137-2149. ISSN: 15728145.

[109] Kabak, M., Burmaoğlu, S., & Kazançoğlu, Y. (2012). A fuzzy hybrid MCDM approach for professional selection. Expert Systems with Applications, 39(3), 3516-3525. ISSN: 09574174.

[110] Fetanat, A., & Khorasaninejad, E. (2015). A novel hybrid MCDM approach for offshore wind farm site selection: A case study of Iran. Ocean & Coastal Management, 109, 17-28. ISSN: 09645691.

[111] Liou, J. J., & Chuang, M. L. (2008). A hybrid MCDM model for evaluating the corporate image of the airline industry. International Journal of Applied Management Science, 1(1), 41-54. ISSN: 17558913.

[112] Zolfani, S. H., Esfahani, M. H., Bitarafan, M., Zavadskas, E. K., & Arefi, S. L. (2013). Developing a new hybrid MCDM method for selection of the optimal alternative of mechanical longitudinal ventilation of tunnel pollutants during automobile accidents. Transport, 28(1), 89-96. ISSN: 16484142.

[113] Tavana, M., Zandi, F., & Katehakis, M. N. (2013). A hybrid fuzzy group ANP–TOPSIS framework for assessment of e-government readiness from a CiRM perspective. Information and Management, 50(7), 383-397. ISSN: 0378-7206.

[114] IJzerman, M. J., Van Til, J. A., & Bridges, J. F. (2012). A comparison of analytic hierarchy process and conjoint analysis methods in assessing treatment alternatives for stroke rehabilitation. The Patient-Patient-Centered Outcomes Research, 5(1), 45-56. ISSN: 1178-1653.

[115] Bouyssou, D., & Pirlot, M. (2005). Conjoint measurement tools for MCDM. In Multiple Criteria Decision Analysis: State of the Art Surveys (pp. 73-112). Springer New York. ISBN: 0-387-23067-X.

[116] Scholl, A., Manthey, L., Helm, R., & Steiner, M. (2005). Solving multiattribute design problems with analytic hierarchy process and conjoint analysis: An empirical comparison. European Journal of Operational Research, 164(3), 760-777. ISSN: 03772217.

[117] Peniwati, K. (2007). Criteria for evaluating group decision-making methods. Mathematical and Computer Modelling, 46(7), 935-947. ISSN: 08957177.

[118] Luce, R. D., & Tukey, J. W. (1964). Simultaneous conjoint measurement: A new type of fundamental measurement. Journal of mathematical psychology, 1(1), 1-27. ISSN: 10960880.

[119]Krantz, D. H. (1964). Conjoint measurement: The Luce-Tukey axiomatization and some extensions. Journal of Mathematical Psychology, 1(2), 248-277. ISSN: 10960880

[120]Tversky, A. (1967). A general theory of polynomial conjoint measurement. Journal of Mathematical Psychology, 4(1), 1-20. ISSN: 10960880.

[121] Simon, H. A. (1957). Models of man: social and rational; mathematical essays on rational human behavior in society setting. Wiley. ASIN: B0007ELZJ4.

[122] Hoffman, P. J. (1967). Cue-consistency and configurality in human judgment. Oregon Research Institute.

[123] Churchman, C. W., & Churchman, C. W. (1961). Prediction and optimal decision: Philosophical issues of a science of values. Englewood Cliffs, NJ: Prentice-Hall. ISBN: 9780313234187.

[124] Raghavarao, D., Wiley, J. B., & Chitturi, P. (2010). Choice-based conjoint analysis: models and designs. CRC Press. ISBN: 1420099965

[125] Green, P. E., & Rao, V. R. (1971). Conjoint measurement for quantifying judgmental data. Journal of Marketing research, 355-363. ISSN: 15477193.

[126] Green, P. E., & Srinivasan, V. (1990). Conjoint analysis in marketing: new developments with implications for research and practice. Journal of Marketing, 3-19. ISSN: 15477185.

[127] Johnson, R.M. (1987). Adaptive conjoint analysis. In Proceedings of Sawtooth Software Conference on Perceptual Mapping, Conjoint analysis, and Computer Interviewing, Sawtooth Software, Ketcham, 253-265.

[128] Elrod, T., Louviere, J. J., & Davey, K. S. (1992). An empirical comparison of ratings-based and choice-based conjoint models. Journal of Marketing research, 29(3), 368-377. ISSN: 15477193.
[129] Acito, F. (1977). An investigation of some data collection issues in conjoint measurement. Contemporary marketing thought (82-85). Chicago: Proceedings. American Marketing Association.
[130] Scott, J. E., & Wright, P. (1976). Modeling an organizational buyer's product evaluation strategy: Validity and procedural considerations. Journal of Marketing Research, 211-224. ISSN: 15477193.
[131] McCullough, J., & Best, R. (1979). Conjoint measurement: temporal stability and structural reliability. Journal of Marketing Research, 26-31. ISSN: 15477193.
[132] Acito, F., and Olshavsky, R. W. (1981). Limits to accuracy in conjoint analysis. Advances in Consumer Research, 8(1). ISSN: 00989258.
[133] Kuhfeld, W. F. (2005). Marketing research methods in SAS. Experimental Design, Choice, Conjoint, and Graphical Techniques. Cary, NC, SAS-Institute TS-722.
[134]Green, P. E., & Srinivasan, V. (1978). Conjoint analysis in consumer research: issues and outlook. Journal of consumer research, 103-123. ISSN: 00935301.
[135] Deng, H., Yeh, C. H., & Willis, R. J. (2000). Inter-company comparison using modified TOPSIS with objective weights. Computers & Operations Research, 27(10), 963-973. ISSN: 03050548.
[136] Shih, H. S., Shyur, H. J., & Lee, E. S. (2007). An extension of TOPSIS for group decision making. Mathematical and Computer Modelling, 45(7), 801-813. ISSN: 08957177
[137] Klir, G. J., St Clair, U., & Yuan, B. (1997). Fuzzy set theory: foundations and applications. Prentice-Hall, Inc. ISBN: 0131011715.
[138] Phillips, J. K., Klein, G., and Sieck, W. R. (2004). Expertise in judgment and decision making: A case for training intuitive decision skills. Blackwell handbook of judgment and decision making, pp. 297-315. ISBN: 9781405107464.
[139] Eeckloo, K., Van Herck, G., Van Hulle, C., and Vleugels, A. (2004). From Corporate Governance To Hospital Governance: Authority, transparency and accountability of Belgian non-profit hospitals' board and management. Health Policy, Vol. 68 No. 1, pp. 1-15. ISSN: 01688510.
[140] Bebchuk, L. A. (2005). The case for increasing shareholder power. Harvard Law Review, pp. 833-914. ISSN: 0017811X.
[141] Carnegie, G. D., and O'Connell, B. T. (2014). A longitudinal study of the interplay of corporate collapse, accounting failure and governance change in Australia: Early 1890s to early 2000s. Critical Perspectives on Accounting, Vol. 25 No. 6, pp. 446-468. ISSN: 10959955.

[142] Tricker, R. I. (2015). Corporate governance: Principles, policies, and practices. Oxford University Press, USA. ISBN-13: 978-0199607969.
[143] Joseph, J., Ocasio, W., and McDonnell, M. H. (2014). The structural elaboration of board independence: Executive power, institutional logics, and the adoption of CEO-only board structures in US corporate governance. Academy of Management Journal, Vol. 57 No. 6, pp. 1834-1858. ISSN: 00014273.
[144] Markham, J. W. (2015). A financial history of modern US corporate scandals: From Enron to reform. Routledge. ISBN-13: 978-0765615831.
[145] Soltani, B. (2014). The anatomy of corporate fraud: A comparative analysis of high profile American and European corporate scandals. Journal of business ethics, Vol. 120 No. 2, pp. 251-274. ISSN: 15730697.
[146] Furrer, O., Rajendran Pandian, J., and Thomas, H. (2007). Corporate strategy and shareholder value during decline and turnaround. Management Decision, Vol. 45 No. 3, pp. 372-392. ISSN: 00251747.
[147] Judge, W. Q., Gaur, A., and Muller-Kahle, M. I. (2010). Antecedents of shareholder activism in target firms: evidence from a multi-country study. Corporate Governance: An International Review, Vol. 18 No. 4, pp. 258-273. ISSN: 14678683.
[148] Raviv, A. (2011). Shareholders vs. Management: Split Decision. Kellogg School of Management, available at: http://insight.kellogg.northwestern.edu/article/
[149] Rust, R. T., & Zahorik, A. J. (1993). Customer satisfaction, customer retention, and market share. Journal of retailing, 69(2), 193-215. ISSN: 00224359.
[150] Elrod, T., Johnson, R. D., & White, J. (2004). A new integrated model of noncompensatory and compensatory decision strategies. Organizational Behavior and Human Decision Process, 95, 1–19. ISSN: 10959920.
[151] Kruskal, J. B. (1965). Analysis of factorial experiments by estimating monotone transformations of the data. Journal of the Royal Statistical Society, Series B (Methodological), pp. 251-263. ISSN: 13697412.
[152] Tervonen, T., Figueira, J. R., Lahdelma, R., Dias, J. A., & Salminen, P. (2009). A stochastic method for robustness analysis in sorting problems. European Journal of Operational Research, 192(1), 236-242. ISSN: 03772217.
[153] Zardari, N. H., Ahmed, K., Shirazi, S. M., & Yusop, Z. B. (2014). Weighting Methods and Their Effects on Multi-Criteria Decision Making Model Outcomes in Water Resources Management. Springer. ISBN 978-3-319-12586-2.
[154] Hobbs, B. F. (1980). A comparison of weighting methods in power plant siting*. Decision Sciences, 11(4), 725-737.
[155] Hajkowicz, S. A., McDonald, G. T., & Smith, P. N. (2000). An evaluation of multiple objective decision support weighting techniques in natural

resource management. Journal of Environmental Planning and Management, 43(4), 505-518. ISSN: 09640568.
[156] Eckenrode, R. T. (1965). Weighting multiple criteria. Management science, 12(3), 180-192. ISSN: 15265501.
[157] Nijkamp, P., Rietveld, P., & Voogd, H. (2013). Multicriteria evaluation in physical planning. Elsevier. ISBN-13: 978-0444881243.
[158] Hobbs, B. F., & Meier, P. (2012). Energy decisions and the environment: a guide to the use of multicriteria methods (Vol. 28). Springer Science & Business Media. ISBN: 079237875X
[159] Voogd, H. (1983). Multicriteria evaluation for urban and regional planning. Taylor & Francis. ISBN-10: 0-85086-106-3.
[160] Rao, V. R. (2014). Applied conjoint analysis (p. 56). New York: Springer. ISBN: 3540877533.
[161] McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior.
[162] Monczka, R.M., Carter, J.R., Markham, W.J., Blascovich, J.D. and Slaight, T.H. (2005). Outsourcing Strategically for Sustainable Competitive Advantage. A Joint Research Study, CAPS Research/A.T. Kearney, Inc., Tempe, AZ/Alexandria, VA
[163] Ghodeswar, B., & Vaidyanathan, J. (2008). Business process outsourcing: an approach to gain access to world-class capabilities. Business Process Management Journal, 14(1), 23-38. ISSN: 14637154.
[164] Simanaviciene, R., & Ustinovichius, L. (2010). Sensitivity analysis for multiple criteria decision making methods: TOPSIS and SAW. Procedia-Social and Behavioral Sciences, 2(6), 7743-7744. ISSN: 1877-0428.
[165] Alinezhad, A., & Amini, A. (2011). Sensitivity analysis of TOPSIS technique: the results of change in the weight of one attribute on the final ranking of alternatives. Journal of Optimization in Industrial Engineering, 23-28.
[166] Pannell, D. J. (1997). Sensitivity analysis of normative economic models: theoretical framework and practical strategies. Agricultural economics, 16(2), 139-152. ISSN: 01695150.
[167] Wolters, W. T. M., & Mareschal, B. (1995). Novel types of sensitivity analysis for additive MCDM methods. European Journal of Operational Research, 81(2), 281-290. ISSN: 03772217.
[168] Mareschal, B. (1991). Weight stability analysis for additive multicriteria methods (No. 2013/9377). ULB--Universite Libre de Bruxelles.
[169] Mareschal, B. (1988). Weight stability intervals in multicriteria decision aid. European Journal of Operational Research, 33(1), 54-64. ISSN: 03772217.

LIST OF AUTHORS PUBLICATIONS

The following section presents a summary of the researcher's publication activities. In Table 43 is a breakdown of the author's number of publications in impacted journals indexed in Web of Science (ISI) and Scopus, together with book chapters and conference papers. In addition is a full list of all the selected publications.

The following is the list of selected publications in reference format.

Complete overview

Table 43: Publications as at 05.12.2015

Impacted journals	5
Scopus Journals	1
Conference papers	9

Impacted journals:

[1] AFFUL-DADZIE, E., AFFUL-DADZIE, A., KOMÍNKOVÁ OPLATKOVÁ, Z. Measuring progress of the millennium development goals: A fuzzy comprehensive evaluation approach. Applied Artificial Intelligence: an international journal, 2014, vol. 28, no. 1, p. 1-15. ISSN 0883-9514.

[2] AFFUL-DADZIE, A., AFFUL-DADZIE, E., NABARESEH, S., KOMÍNKOVÁ OPLATKOVÁ, Z. Tracking progress of African Peer Review Mechanism (APRM) using fuzzy comprehensive evaluation method. Kybernetes, 2014, vol. 43, no. 8, p. 1193-1208. ISSN 0368-492X.

[3] AFFUL-DADZIE, E., NABARESEH, S., AFFUL-DADZIE, A., KOMÍNKOVÁ OPLATKOVÁ, Z. A fuzzy TOPSIS framework for selecting fragile states for support facility. Quality and Quantity: International Journal of Methodology, 2014, vol. 48, no. 6, p. -. ISSN 0033-5177.

[4] AFFUL-DADZIE, A., AFFUL-DADZIE, E. A TOPSIS Extension Framework for Re-Conceptualizing Sustainability Measurement. Kybernetes, 2015, vol. 45, no. 1. ISSN 0368-492X.

[5] AFFUL-DADZIE, E., NABARESEH, S., KOMÍNKOVÁ OPLATKOVÁ, Z., KLÍMEK, P. Model for Assessing Quality of Online Health Information: A Fuzzy VIKOR Based Method. Journal of Multi-Criteria Decision Analysis, 2015. ISSN: 1099-1360.

[6] AFFUL-DADZIE, E., AFFUL-DADZIE, A., KOMÍNKOVÁ OPLATKOVÁ, Z. Assessing Commercial Viability of Technology Start-up Businesses in a Government Venture Capital under Fuzzy Intuitionistic Environment. International Journal of Fuzzy Systems, 2016. ISSN: 1562-2479.

Conference papers

[7] AFFUL-DADZIE, E., NABARESEH, S., KOMÍNKOVÁ OPLATKOVÁ, Z., KLÍMEK, P. Ranking Fragile States for Support Facility: A Fuzzy TOPSIS Approach. In The 2014 10th International Conference on Natural Computation. New Jersey, Piscataway: IEEE, 2014, p. 263-269. ISBN 978-1-4799-5150-5.

[8] AFFUL-DADZIE, E., NABARESEH, S., KOMÍNKOVÁ OPLATKOVÁ, Z. Patterns and Trends in the Concept of Green Economy: A Text Mining Approach. In Advances in Intelligent Systems and Computing. 285. Heidelberg: Springer-Verlag Berlin, 2014, p. 143-154. ISSN 2194-5357. ISBN 978-3-319-06739-1.

[9] AFFUL-DADZIE, E., NABARESEH, S., KOMÍNKOVÁ OPLATKOVÁ, Z., KLÍMEK, P. Enterprise Competitive Analysis and Consumer Sentiments on Social Media: Insights from Telecommunication Companies. In DATA 2014 3rd International Conference on Data Management Technologies and Applications. Wien : Springer Wien, 2014, p. 22-32. ISBN 978-989-758-035-2.

[10] AFFUL-DADZIE, E., NABARESEH, S., KOMÍNKOVÁ OPLATKOVÁ, Z. Fuzzy VIKOR Approach: Evaluating Quality of Internet Health Information. In Federated Conference on Computer Science and Information Systems. New Jersey, Piscataway : IEEE, 2014, p. 191-198. ISBN 978-83-60810-57-6.

[11] AFFUL-DADZIE, E. PERFORMANCE EVALUATION OF THE MILLENNIUM DEVELOPMENT GOALS USING FUZZY COMPREHENSIVE EVALUATION METHOD. In MENDEL 2013 19th International Conference on Soft Computing. Brno : Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2013, p. 265-272. ISSN 1803-3814. ISBN 978-80-214-4755-4.

[12] NABARESEH, S., AFFUL-DADZIE, E., KLÍMEK, P. Security of electronic transactions in developing countries: a cluster and decision tree mining approach. In Proceedings of ICIME 2015. Academic Conference and Publishing International Limited, 2015, ISBN 978-1-910810-08-8.

[13] AFFUL-DADZIE, E., KOMÍNKOVÁ OPLATKOVÁ, Z., NABARESEH, S. Selecting start-up businesses in a public Venture capital financing using Fuzzy PROMETHEE. In 19th International Conference on Knowledge-Based and Intelligent Information & Engineering Systems, Procedia Computer Science, 60, 63-72, 2015, ISSN: 1877-0509.

[14] AFFUL-DADZIE, E., KOMÍNKOVÁ OPLATKOVÁ, Z., NABARESEH, S. Selecting Start-up Businesses in a Public Venture Capital with Intuitionistic Fuzzy TOPSIS. In proceedings of International Conference on Soft Computing and Applications, 2015.

[15] NABARESEH, S., AFFUL-DADZIE, E., KOMÍNKOVÁ OPLATKOVÁ, Z., KLÍMEK, P. Selecting Countries for Developmental Aid Programs using Fuzzy PROMETHEE. In proceedings of SAI Intelligent Systems Conference, 2015.

Curriculum Vitae

Personal information

First name / Surname	**Eric Afful-Dadzie**
Address	Nam. T.G Masaryka 3050, 76001-Zlin
Telephone(s)	+420 777 546 790 (personal)
Email(s)	paadadzie2002@gmail.com (personal) afful@fai.utb.cz (work)
Nationality	Ghanaian
Date of birth	20. 09. 1981

Education and training

Dates	2012 – present
Title of qualification awarded	Ph.D.
Principal subjects/occupational skills covered	Engineering Informatics
Name and type of organisation providing education and training	Tomas Bata University in Zlín, Faculty of Applied Informatics

Education and training

Dates	2010 – 2012
Title of qualification awarded	Ing. (M.Sc.)
Principal subjects/occupational skills covered	Systems Engineering and Informatics
Name and type of organisation providing education and training	Czech University of Life Sciences Prague (Česká zemědělská univerzita v Praze)

Education and training

Dates	2002 – 2006
Title of qualification awarded	B.Ed.
Principal subjects/occupational skills covered	Computer Science
Name and type of organisation providing education and training	University of Cape Coast - Ghana

Education and training

Dates: 1997 – 1999

Title of qualification awarded: Senior Secondary School Certificate Examination

Principal subjects/occupational skills covered: Elective Science

Name and type of organisation providing education and training: St. John’s School, Sekondi - Ghana

Personal skills and competences

Mother tongue: **Fante**

Other language(s)
Self-assessment

English

Understanding		Speaking		Writing
Listening	Reading	Spoken interaction	Spoken production	
Advanced	Advanced	Pre-advanced	Intermediate	Advanced